Optimal impulsive control of coffee berry borers in a berry age-structured epidemiological model

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Abstract

The coffee berry borer (CBB) *Hypothenemus hampei* (Coleoptera: Scolytidae) is the most important insect pest affecting coffee production worldwide and generating huge economic losses. As most of its life cycle occurs inside the coffee berry, its control is extremely difficult. To tackle this issue, we solve an optimal control problem based on a berry age-structured dynamical model that describes the infestation dynamics of coffee berries by CBB during a cropping season. This problem consists in applying a bio-insecticide at discrete times in order to maximise the economic profit of healthy coffee berries, while minimising the CBB population for the next cropping season. We derive analytically the first-order necessary optimality conditions of the control problem. Numerical simulations are provided to illustrate the effectiveness of the optimal control strategy.

Keywords

Plant-pest interactions; PDE dynamical model; Impulsive control; Maximum principle; Numerical simulations; *Hypothenemus hampei*

I INTRODUCTION

Coffee berry borer (CBB), *Hypothenemus hampei* (Coleoptera: Scolytidae) is the most damaging insect pest of coffee worldwide, affecting both the yield and quality of coffee products [1, 5], and causing more than US$500 million in damages annually [6]. The CBB life cycle is composed of five distinct stages: egg, larva, prepupa, pupa and adult [11]. It starts when an adult (fertilised) female emerges from an infested berry to seek a new host. It selects and bores a hole in a uninfested berry. It lay its eggs in internal galleries and remains inside the coffee berry after oviposition until it dies. After hatching, the larvae feed on the coffee seeds. Young females mate with their male siblings inside the berry. Male CBB do not fly and remain inside the berry [8]. Fertilised females emerge from the infested berry, and the loop is closed. The choice of a new host by colonising females depends mainly on the age of the coffee berry. CBB attack immature and mature coffee berries from three months after flowering up to harvest period, with a preference for older berries [9].

In response to CBB attacks, integrated pest management programs for coffee plantation have been developed, which involve several control strategies. Chemical control consists in applying synthetic insecticides designed to kill the CBB during the colonisation of young coffee berries [11]. Biological control is based on natural enemies or products derived from living organisms and aims at maintaining CBB at an acceptable level of harmfulness. The main enemies of CBB are parasitic Hymenoptera, certain entomopathogenic fungus and nematodes [5, 11]. Cultural practices consist of various activities, such as sanitary harvesting, which is the elimination of residual berries present on the branches of trees and on the ground [5, 11]. Finally, trapping methods consist in using attractive traps to capture colonising females during their migration flights [7].

The purpose of this work is to optimise CBB control strategies consisting in spraying a biocontrol agent or bio-insecticide at discrete times. It is based on a dynamical model describing the interactions between CBB and coffee berries during a cropping season, which incorporates a berry age structure to account for CBB preference for older berries [15]. To achieve this goal, we formulate and solve an optimal impulsive control problem whose objective is to maximise the profit, while minimising the CBB population for the next cropping season. We previously studied a similar problem in [14], but the model did not include the berry age structure and the control was continuous instead of impulsive. The present paper hence proposes two notable improvements compared to our previous work. Impulsive bio-insecticide spraying, in particular, is more relevant for field implementation.

The remainder of the paper has the following structure. In Section II, we formulate the control problem. In Section III, the first order necessary conditions for optimality are established. Finally, results are illustrated by numerical simulations in Section IV.

II THE MODEL AND CONTROL PROBLEM STATEMENT

2.1 Coffee berry–CBB interaction model

In this study, we consider the epidemiological model proposed in [15] describing the infestation dynamics of coffee berries by CBB. Coffee berries are characterised by their age and their epidemic status: $s(t, a)$ and $i(t, a)$ are respectively the age-specific density of healthy and infested coffee berries at the time t and age $a \in [0, a_\dagger]$, where $a_\dagger$ is the maximal berry age. The CBB population is divided in two groups, the colonising females denoted by $y(t)$, which correspond to the flying fertilised females looking for their host, and infesting females that are laying eggs inside the berries. Although the sex ratio is largely female-biased [5], we assume that there are enough males in each generation to fertilise young females. The infestation dynamics of coffee berries by CBB are described by the following age-structured model:

$$\begin{cases} \partial_t s(t,a) + \partial_a s(t,a) = -\beta(a) f(B,y) s(t,a) - \mu(a) s(t,a), \\ \partial_t i(t,a) + \partial_a i(t,a) = \beta(a) f(B,y) i(t,a) - \nu(a) i(t,a), \\ \dot{y}(t) = -\varepsilon f(B,y) \|\beta s(t,.)\| - \mu_y y(t) + \phi z(t), \\ \dot{z}(t) = \varepsilon f(B,y) \|\beta s(t,.)\| - \mu_z z(t), \end{cases} \tag{1}$$

completed by the following boundary and initial conditions:

$$s(t,0) = g(t), \quad i(t,0) = 0, \quad s(0,a) = s_0(a), \quad i(0,a) = i_0(a). \tag{2}$$

In this model, new healthy berries are produced at time-dependent rate $g(t)$. The colonising CBB females infest the healthy berries at rate $\beta(a)f(B,y)$, where $\beta(.)$ is the berry age-dependent infestation rate and $f(.,.)$ is an interaction function which depends on the total healthy berries $B(t) = \int_0^{a_\dagger} s(t,a)da$ and colonising females y. This creates a berry transfer from healthy $s(t,a)$ to infested berries $i(t,a)$ and simultaneously, from colonising $y(t)$ to infesting females $z(t)$ (with $\|\beta s(t,.)\| = \int_0^{a_\dagger} \beta(a)s(t,a)da$). The scaling parameter ε corresponds to the number of colonising females per infested berry; usually, $\varepsilon = 1$ CBB/berry, since super-parasitism is rarely observed in the plantation [9]. Infesting females lay eggs inside the coffee berries, which go through their development cycle until fertilised adult females emerge at rate ϕ. Colonising and infesting CBB females undergo mortality at rate μ_y and μ_z; healthy and infested berries at the rate $\mu(a)$ and $\nu(a)$. This model relies on the following assumptions.

Assumption 1: Positivity and smoothness of the functions and parameters of system (1)

1. Parameters ϕ, ε, μ_y, μ_z and initial conditions y_0, z_0 are nonnegative.
2. $g(.) \in L^\infty_+(0,\infty)$, $\beta(.) \in L^\infty_+(0,a_\dagger)$; boundary conditions $s_0(.), i_0(.) \in L^1_+(0,a_\dagger)$ and are bounded.
3. Mortality rates $\mu(.), \nu(.) \in L^\infty_+(0,a_\dagger)$, $\nu(a) \geq \mu(a)$; moreover, there exists a real number $\widetilde{\mu} > 0$ satisfying $\mu(a) \geq \widetilde{\mu}$ for almost every $a \in [0,a_\dagger]$.
4. Contact function $f(.,.)$ is bounded and C^1–Lipschitz continuous for both arguments; moreover, $f(B,y)$ decreases with B and increases with y, with $f(B,0) = 0$ and for all $y > 0$, $\lim\limits_{B\to+\infty} f(B,y)B$ is finite.

$(L^1_+(I), \|.\|)$ is the space of nonnegative measurable functions $L^1(I)$ equipped by the product norm and $(L^\infty_+(I), \|.\|_\infty)$ the space of nonnegative functions $L^\infty(I)$ over the set $I \subset \mathbb{R}$.

We then include in this model the possibility to apply a bio-insecticide at discrete times in the coffee plantation. The bio-insecticide can for instance be based on the entomopathogenic fungus *Beauveria bassiana* [13]. It prevents berry infestation: when a colonising CBB bores its hole in a healthy berry and comes into contact with the bio-insecticide, it gets killed by the biocontrol agent. To implement this control in the model, we introduce a new state variable $v(t)$, which corresponds to the bio-insecticide load in the plantation at time t. Note that variable $i(t,a)$, representing infested berries, does not appear in the remaining equations of system (1) and is not affected by our control. Therefore, one can drop the $i(t,a)$–equation to control the system dynamics. We then obtain the following controlled system:

$$\begin{cases} \partial_t s(t,a) + \partial_a s(t,a) = -\beta(a) q(v(t)) f(B,y) s(t,a) + \mu(a) s(t,a), \\ \qquad \dot{y}(t) = -\varepsilon f(B,y) \|\beta s(t,.)\| - \mu_y y(t) + \phi z(t), \\ \qquad \dot{z}(t) = \varepsilon q(v(t)) f(B,y) \|\beta s(t,.)\| - \mu_z z(t), \\ \qquad \dot{v}(t) = -\gamma v(t), \qquad\qquad \text{for } t \neq t_n, \\ \quad \Delta v(t_n) = v(t_n^+) - v(t_n) = h_n \quad \text{for } t = t_n, \\ \quad s(t,0) = g(t), \quad s(0,a) = s_0(a), \quad y(0) = y_0, \quad z(0) = z_0, \quad v(0^+) = v_0. \end{cases} \tag{3}$$

The control $h_n \geq 0$ is applied periodically in the plantation at discrete times $t_n = n\tau$, where $n \in \{0, 1, 2, \dots, N_f\}$ and τ is the application period, *i.e.* the time elapsed between two successive applications. The t_n^+ notation depicts the instant just after t_n, so $v(t_n^+) = \lim_{j \to 0^+} v(t_n + j)$ is the bio-insecticide load instantly after the control application. The bio-insecticide load persists in the plantation and decays with rate γ. Its initial condition $v(0^+)$ is nonnegative. As in [14], function $q(.) : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ represents the modulation of the CBB infestation rate by the bio-insecticide as follows: $q(v) = 1 - \frac{\xi v}{v+k}$, where $\xi \in (0,1)$ denotes the maximal effectiveness of the bio-insecticide load and the positive constant k the load half saturation.

2.2 Optimal control problem

We now formulate the optimal control problem. The main goal of coffee growers is to produce good quality coffee berries, that can be sold at a high price, with low control costs, so as to maximise their profit. They also aim at reducing the CBB population at the end of the cropping season, to limit the risk for the next season. We assume that the harvest takes place at the end of the cropping season, denoted by t_f. We also assume that (almost) all berries, and hence (almost) all infesting females that are inside berries, are picked at t_f , so that only colonising females (y) remain in the plantation after harvest. Lastly, we assume that infested coffee berries have a negligible monetary value, so that the yield consists only of healthy berries. We define the class of admissible controls as:

$$\mathcal{M} := \{h = (h_n) \in \mathbb{R}^{N_f+1} : 0 \leq h_n \leq h_{\max}, \forall n \in \{0, 1, 2, \dots N_f\}\},$$

where $h_{\max}$ represents the upper bound of control. The objective functional is defined for control $h = (h_n)_{n \in \{0,1,2,\dots N_f\}} \in \mathcal{M}$ as follows:

$$\mathcal{J}(h) = \int_0^{a_\dagger} \Theta(a) s(t_f, a) da - \sum_{n=0}^{N_f} C(h_n) - C_y y(t_f), \tag{4}$$

subject to the impulsive evolution System (3). The first term is the coffee berry yield, where function $\Theta(.)$ denotes the price of healthy berries, a bounded, continuous and increasing function of the berry age. The second term represents the control cost, where $C(.)$ is a continuous function of the control h_n. The last term is a penalty on the CBB population that remains in the plantation after harvest, weighted by constant C_y.

Our purpose is to maximise the objective functional over the admissible class of controls, *i.e.* seek $h^\star = (h_n^\star)_{n \in \{0,1,2,\dots N_f\}}$ that belongs to $\mathcal{M}$ so that:

$$\mathcal{J}(h^\star) = \max_{h \in \mathcal{M}} \mathcal{J}(h). \tag{5}$$

III OPTIMALITY CONDITIONS

We first establish the existence of an optimal solution to problem (5) in Theorem 1 (see proof in Appendix 1.1) and then characterise the solution using the maximum principle method.

Theorem 1:
There exists an optimal control application $h^\star = (h_n^\star)_{n\in\{0,1,2,\dots N_f\}}$ that belongs to $\mathcal{M}$, which maximises the objective functional $\mathcal{J}(.)$ subject to system (3).

Using the framework of [4] and [3], we derive the optimal control from a combination of the state and the adjoint variables. We determine the adjoint equations by first introducing the sensitivity functions. Let us denote $p = (s, y, z, v)$ and define the solution map: $h \longrightarrow p(h)$. The sensitivity functions are defined by the Gâteaux derivatives: for $\bar{h} \in \mathbb{R}^{N_f+1}$, $(\lambda_s, \lambda_y, \lambda_z, \lambda_v) = \lim_{\epsilon\to 0^+} \epsilon^{-1}\left[p(h+\epsilon\bar{h}) - p(h)\right]$. Consequently, the sensitivity functions $\lambda_s(t,a)$, $\lambda_y(t)$, $\lambda_z(t)$ and $\lambda_v(t)$, corresponding to the state variables $s(t,a)$, $y(t)$, $z(t)$ and $v(t)$, satisfy the following equations:

$$\begin{cases} \partial_t\lambda_s + \partial_a\lambda_s = -q(v)[\beta(a)f(B,y)\lambda_s + \beta s f_B(B,y)\|\lambda_s\| + \beta s f_y(B,y)\lambda_y] \\ \qquad\qquad\qquad + q_v(v)f(B,y)\beta s\lambda_v - \mu\lambda_s, \\ \dot{\lambda}_y = \phi\lambda_z - \varepsilon f(B,y)\|\beta\lambda_s\| - \varepsilon\|\beta s\|f_y(B,y)\lambda_y - \varepsilon\|\beta s\|f_B(B,y)\|\lambda_s\| - \mu_y\lambda_y, \\ \dot{\lambda}_z = q(v)[\varepsilon f(B,y)\|\beta\lambda_s\| + \varepsilon\|\beta s\|f_y(B,y)\lambda_y + \varepsilon\|\beta s\|f_B(B,y)\|\lambda_s\|] \\ \qquad - q_v(v)\varepsilon f(B,y)\|\beta s\|\lambda_v - \mu_z\lambda_z, \end{cases} \tag{6}$$

and:

$$\begin{cases} \dot{\lambda}_v = -\gamma\lambda_v & \text{for } t \neq t_n, \\ \lambda_v(t_n^+) = \lambda_v(t_n) + \bar{h}_n & \text{for } t = t_n,\ n = 0,1,2,\dots N_f. \end{cases} \tag{7}$$

Functions $f_y(.,.)$, $f_B(.,.)$ and $q_v(.)$ represent the partial derivatives of functions $f(.,.)$ and $q(.)$ with respect to their arguments y, B and v. The sensitivity functions respect the following initial and boundary conditions:

$$\lambda_s(t,0) = 0, \quad \lambda_s(0,a) = 0, \quad \lambda_y(0) = \lambda_z(0) = \lambda_v(0) = 0. \tag{8}$$

Next, we introduce $\Phi_s(t,a)$, $\Phi_y(t)$, $\Phi_z(t)$ and $\Phi_v(t)$, the adjoint variables corresponding to the state variables $s(t,a)$, $y(t)$, $z(t)$ and $v(t)$ respectively. The adjoint equations are derived by using the adjoint operator associated with the sensitivity equations (6,7), together with appropriate tranversality and boundary conditions. The adjoint equations are given by:

$$\begin{cases} \partial_t\Phi_s + \partial_a\Phi_s = q(v)[\beta f(B,y)(\Phi_s - \varepsilon\Phi_z) + f_B(B,y)(\|\beta s\Phi_s\| - \varepsilon\|\beta s\|\Phi_z)] \\ \qquad\qquad\qquad + [\varepsilon\beta f(B,y) + \varepsilon\|\beta s\|f_B(B,y)]\,\Phi_y + \mu\Phi_s, \\ \dot{\Phi}_y = q(v)f_y(B,y)\left[-\varepsilon\|\beta s\|\Phi_z + \|\beta s\Phi_s\|\right] + \varepsilon\|\beta s\|f_y(B,y)\Phi_y + \mu_y\Phi_y, \\ \dot{\Phi}_z = -\phi\Phi_y + \mu_z\Phi_z, \\ \dot{\Phi}_v = q_v(v)f(B,y)\left[-\|\beta s\Phi_s\| + \varepsilon\|\beta s\|\Phi_z\right] + \gamma\Phi_v, \end{cases} \tag{9}$$

and:

$$\Phi_v(t_n^+) = \Phi_v(t_n) \quad \text{for } n = 0,1,2,\dots N_f, \tag{10}$$

with the following tranversality conditions:

$$\Phi_s(t_f, a) = \Theta(a), \quad \Phi_s(t, a_\dagger) = 0, \quad \Phi_y(t_f) = -C_y, \quad \Phi_y(t_f) = \Phi_v(t_f) = 0. \tag{11}$$

We obtain the following result about the characterisation of the optimal control strategy.

Theorem 2:
For any optimal control application $h = (h_n)_{n\in\{0,1,2,\ldots N_f\}} \in \mathcal{M}$, then the Gâteaux derivative of $\mathcal{J}(.)$ is $D_{\bar{h}}\mathcal{J}(h) = \sum_{n=0}^{N_f} \left[\Phi_v(t_n^+) - C'(h_n)\right] \bar{h}_n$, where $C'(.)$ represents the derivation of function $C(.)$ with respect to its argument $\bar{h}_n$ and $\bar{h} \in \mathcal{V}_h$, with the set $\mathcal{V}_h := \{\bar{h} \in \mathbb{R}^{N_f+1} : \exists \epsilon \in [0,1];\ h + \epsilon\bar{h} \in \mathcal{M}\}$.

The method used to prove Theorem 2 is the impulsive maximum principle (see proof in Appendix 1.2), which gives the first order necessary optimality conditions [3, 4], but does not provide an efficient way to compute the optimal control.

We now suppose that the control cost is $C(h_n) = \eta h_n^2$, so that the objective functional (4) is quadratic in control. For this particular cost function, Theorem 3 provides a means for computing the optimal control strategy (see proof in Appendix 1.3).

Theorem 3:
There exists an optimal control $h^\star = (h_n^\star)_{n\in\{0,1,2,\ldots N_f\}} \in \mathcal{M}$, with corresponding optimal states $s^\star(t,a)$, $y^\star(t)$, $z^\star(t)$ and $v^\star(t)$, maximising the objective functional $\mathcal{J}(.)$ defined in (4). Moreover, the characterisation of the optimal control $h^\star$ is given by:

$$h_n^\star = \max\left\{0, \min\left\{\frac{\Phi_{v^\star}(t_n^+)}{2\eta}, h_{\max}\right\}\right\}, \quad \forall n \in \{0, 1, 2, \ldots N_f\}. \tag{12}$$

IV NUMERICAL SIMULATIONS

We present numerical simulations illustrating the effect of the optimal control strategy on the infestation process of coffee berries by CBB in the plantation. Some system parameters, in particular those associated with the development cycle of the CBB and the infestation rate are estimated from the literature [11]. More information is available in [12, 14, 15]. Our numerical approach is based on an extension of the forward-backward sweep method [10], originally proposed to solve the optimal control of continuous ordinary differential equations with impulsive controls. The procedure consists of the following steps: first, the state equations in (3) are solved using the forward semi-implicit finite difference in time and backward difference in age, with an initial guess for the control variable. Second, the adjoint equations in (9) are solved by backward semi-implicit finite difference in time and forward difference in age, using the solutions of the state equations. Next, the control is updated with the new values of the state and adjoint solutions given by Theorem 3. The algorithm is repeated until the states and control converge.

In the simulations, a constant berry production rate $g(t) \equiv g$ and an age-independent berry mortality rate $\mu(a) \equiv \mu$ are chosen. Furthermore, the infestation rate is defined by the following function:

$$\beta(a) = \begin{cases} \beta_{\min} & 0 \le a < a_\beta, \\ \beta_{\min} + \beta_a(1 - e^{-k_\beta(a - a_\beta)}) & a_\beta \le a \le a_\dagger, \end{cases} \tag{13}$$

its average value being $\bar{\beta} = \frac{1}{a_\dagger}\int_0^{a_\dagger}\beta(a)da$. The CBB–berry interaction function f is modelled by $f(B,y) = \frac{y}{y+\alpha B+1}$. More details on these functions are given in [15]. To apply Theorem 3, we use a quadratic control cost $C(h_n) = \eta h_n^2$. Moreover, we choose a sigmoid function to model the price of healthy coffee berries according to berry age:

$$\Theta(a) = \frac{\overline{\Theta}a^m}{a^m + a_\Theta^m}, \tag{14}$$

where $\overline{\Theta}$ is the asymptotic price of healthy coffee berries, a_Θ is the age at which berries are at half asymptotic price and $m \in \mathbb{N}^\star$ is the Hill constant. Note that with a finite age bounded by $a_\dagger$, the maximum price that mature berries can reach is $\frac{\overline{\Theta}a_\dagger^m}{a_\dagger^m + a_\Theta^m}$.

Moreover, we use the following initial conditions. At the beginning of the cropping season, flowering has not started yet. We also assume that there are only colonising females, as infesting females were eliminated from the plantation when berries were picked during the preceding harvest. So there are initially neither coffee berries nor infesting females, *i.e.* $s_0(a) = 0$ for all $a \in [0, a_\dagger]$ and $z(0) = 0$. The number of initial colonising females is set at $y(0) = 10^4$ females. We also assume that the control is absent in the plantation so the initial load is $v(0) = 0$.

We consider two periods for the bio-insecticide application in the plantation during the cropping season in Figures 1 and 2: $\tau = 7$ days (1 week, red curves) in and $\tau = 14$ days (2 weeks, magenta curves). All parameter values are given in Table 1.

In Figure 1, panels (a–d) show the dynamics of the state variables, panel (e) the optimal bio-insecticide application; panel (f) represents the effect of bio-insecticide load on the infestation rate, that is $\sigma(v) = 1 - q(v) = \frac{\xi v}{k+v}$. Trajectories of colonising and infesting females increase, both in the uncontrolled (dashed blue curves) and controlled (red and magenta curves) case. Due to the CBB preference for older coffee berries, this growth starts slowly. In all cases, the bio-insecticide (panel (e)) is applied at its maximal value at the beginning of the cropping season and decreases progressively. Its effect (panel (f)) rapidly increases but remains below its maximum value ($\xi = 0.8$), which denotes a trade-off between the control effectiveness and its cost. The colonising (panel (b)) and infesting (panel (c)) female trajectories are much lower when the optimal control is applied (red and magenta plain curves) than in the case without control (dashed blue curve). Consequently, the healthy coffee berries (panel (a)) are notably higher with control at the end of cropping season. The control is more effective when applied weekly than every two weeks (Figure 1), both in terms of healthy berries and colonising females at the end of the cropping season. However, this result could change if the control cost function C in the objective functional (4) was modified, for instance if the bio-insecticide cost η was higher or if C included fixed labour costs for each application.

Figure 2 illustrates the evolution of the final berry density and price of coffee berries as functions of the berry age at harvest for bio-insecticide applications every week (red curves) and every 2 weeks (magenta curves). We observe a strong decrease of the mature coffee berries (≥ 90 days) without and with control (left panel) due to the berry age preference of CBB. However, the price distribution of berries increases with berry age but remains considerably lower than in the case without CBB (right panel). Without control (blue curves), one can observe the strong impact of the infestation rate (blue shaded area), as well as the effect of CBB preference for older berries: with the same average value for the infestation rate, the final berry density is higher with (dashed curve) than without (plain curve) age preference. Indeed, in the former case, young berries have a better chance to remain healthy.

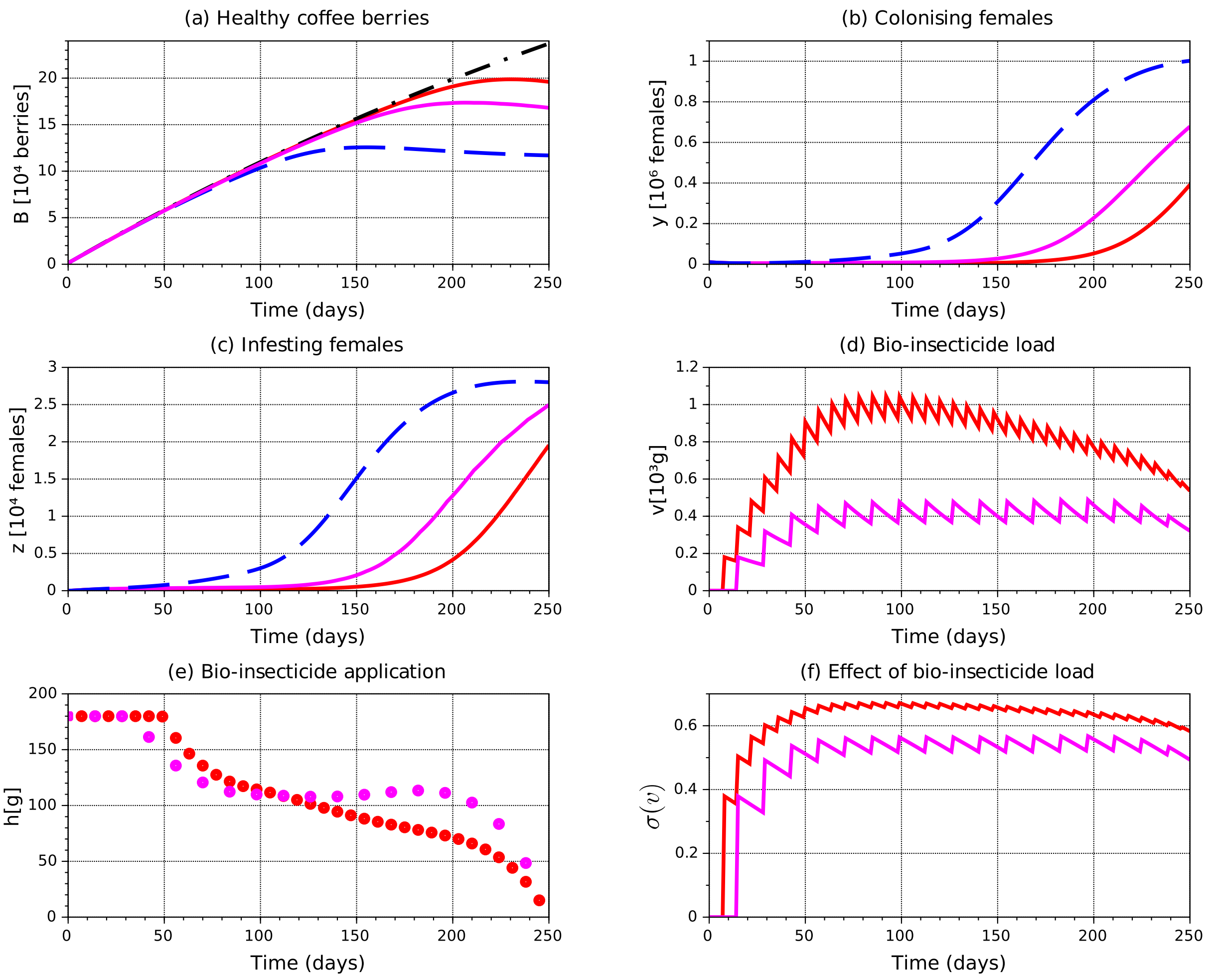

Figure 1: Simulation of system (3) without control (dashed blue curves) and with weekly (plain red curves) and every 2 weeks (plain magenta curves) optimal bio- insecticide applications. The healthy berry growth without pest is also represented in (a) (dash-dotted black curve).

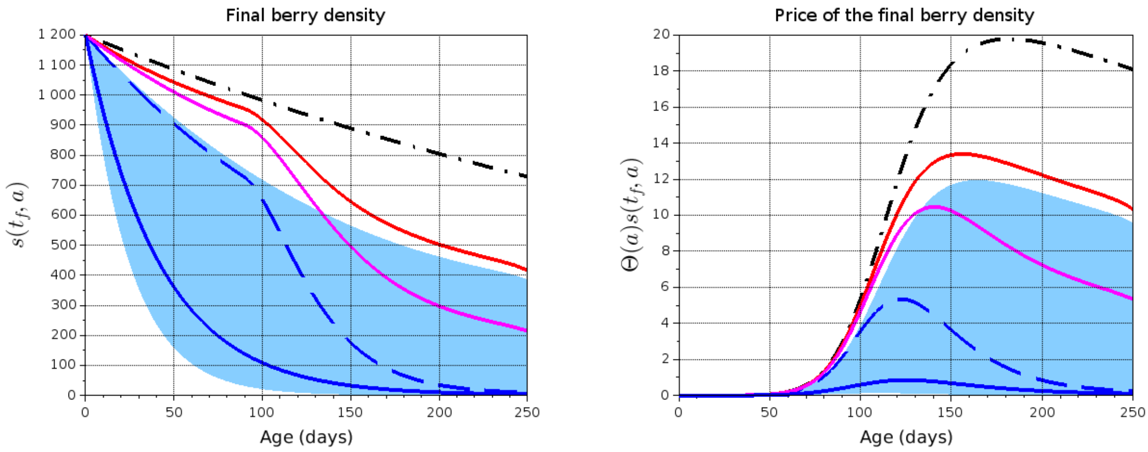

Figure 2: Age distribution of the healthy coffee berries and their price (yield) at the end of the simulation ($t = t_f$) with optimal bio-insecticide application every week (plain red curves) or every 2 weeks (plain magenta curves), without control (dashed blue curves), and without pest (dash-dotted black curves). Cases without control and with various constant age-independent infestation rates β are also represented (blue shaded area delimited by $\beta_{\min}$ and $\beta_{\max} = \beta_{\min} + \beta_a$, plain blue curve for average value $\bar{\beta}$).

V CONCLUSION

In view of the extent of damages caused by CBB in coffee plantations, we aimed in this work at controlling the infestation dynamics of coffee berries by CBB. The control is based on the application of a bio-insecticide at discrete and periodic times, to prevent CBB from infesting healthy berries. The controlled model we used is a semi-discrete system with a continuous berry age structure to represent the CBB preference for older berries. An optimisation problem was formulated in order to maximise the yield of healthy coffee berries, while minimising the cost of control, as well as the remaining CBB population for the next cropping season. We showed the existence of an optimal control and gave its characterisation using the maximum principle. Numerical simulations confirmed that the application of the bio-insecticide effectively controls the CBB and considerably increases the yield and profit at the end of cropping season. Weekly applications were found to be more efficient than applications every two weeks, both in terms of CBB population reduction and profit maximisation.

REFERENCES

References

[1] R. H. Le Pelley. *Pests of Coffee*. Tropical science series. London, UK: Longmans, 1968.

[2] V. Lakshmikantham, D. D. Bainov, and P. S. Simeonov. *Theory of Impulsive Differential Equations*. Volume 6. Series in Modern Applied Mathematics. World Scientific, 1989.

[3] J. Yong and P. Zhang. “Necessary conditions of optimal impulse controls for distributed parameter systems”. In: *B. Aust. Math. Soc.* 45.2 (1992), pages 305–326.

[4] J. Zabczyk. *Mathematical control theory: An introduction*. Systems & Control: Foundations & Applications. Boston: Birkhäuser, 1995.

[5] A. Damon. “A review of the biology and control of the coffee berry borer, Hypothenemus hampei (Coleoptera: Scolytidae)”. In: *Bull. Entomol. Res.* 90.6 (2000), pages 453–465.

[6] F. E. Vega, E. Rosenquist, and W. Collins. “Global project needed to tackle coffee crisis”. In: *Nature* 425.6956 (2003), page 343.

[7] R. Messing. “The Coffee Berry Borer (Hypothenemus hampei) Invades Hawaii: Preliminary Investigations on Trap Response and Alternate Hosts”. In: *Insects* 3.3 (2012), pages 640–652.

[8] W. D. Silva, G. M. Mascarin, E. M. Romagnoli, and J. M. S. Bento. “Mating Behavior of the coffee berry borer, Hypothenemus hampei (Ferrari) (Coleoptera: Curculionidae: Scolytinae)”. In: *J. Insect. Behav.* 25 (2012), pages 408–417.

[9] D. Rodriguez, J. R. Cure, A. P. Gutierrez, J. M. Cotes, and F. Cantor. “A coffee agroecosystem model: II Dynamics of coffee berry borer”. In: *Ecol. Model.* 248 (2013), pages 203–214.

[10] R. Leander, S. Lenhart, and V. Protopopescu. “Optimal control of continuous systems with impulse controls”. In: *Optim. Contr. Appl. Met.* 36.4 (2015), pages 535–549.

[11] L. F. Aristizàbal, A. E. Bustillo, and S. P. Arthurs. “Integrated Pest Management of Coffee Berry Borer: Strategies from Latin America that Could Be Useful for Coffee Farmers in Hawaii”. In: *Insects* 7.1 (2016), page 6.

[12] Y. Fotso, F. Grognard, B. Tsanou, and S. Touzeau. “Modelling and control of coffee berry borer infestation”. In: *14th African Conference on Research in Computer Science and Applied Mathematics*. Stellenbosch, South Africa, Oct. 2018.

[13] E. Greco and M. Wright. "Efficacy of Beauveria bassiana applications on coffee berry borer across an elevation gradient in Hawaii". In: *Biocontrol. Sci. Techn.* 28.11 (2018), pages 995–1013.

[14] Y. Fotso Fotso, S. Touzeau, B. Tsanou, S. Bowong, and F. Grognard. "Modelling and optimal strategy to control coffee berry borer". In: *Math. Methods Appl. Sci.* 44.18 (2021), pages 14569–14592.

[15] Y. Fotso Fotso, S. Touzeau, B. Tsanou, F. Grognard, and S. Bowong. "Mathematical modelling of a pest in an age-structured crop model: The coffee berry borer case". In: *Appl. Math. Model.* 110 (2022), pages 193–206.

A APPENDIX

1.1 Proof of Theorem 1

Let $h_m = \max\{h_n, \forall n = 0, 1, 2, \dots N_f\}$, we have the inequality $v(t_n^+) \leq v(t_n) + h_m$. Let $w(t)$ be the solution of the impulsive system

$$\begin{cases} \dot{w}(t) = -\gamma w(t) & \text{for } t \neq t_n, \\ \Delta w(t_n) = w(t_n^+) - w(t_n) = h_m, & \text{for } t = t_n,\ n = 0, 1, 2 \dots \end{cases}$$

Then, for $t \in (t_n^+, t_{n+1})$, we have from the first equation of system above $w(t) = w(t_n^+)e^{-\gamma(t-t_n)}$. Therefore we get $w(t_{n+1}^+) = w(t_n^+)e^{-\gamma\tau} + h_m$. As $e^{-\gamma\tau} < 1$, we deduce that the sequence $(w(t_n^+))_n$ converges to a fixed point, that is $w(t_n^+) \to \frac{h_m}{1-e^{-\gamma\tau}}$. Finally, we obtain the following periodic solution of $w(t) = \frac{h_m e^{-\gamma(t-t_n)}}{1-e^{-\gamma\tau}} \leq \frac{h_m}{1-e^{-\gamma\tau}}$. Therefore, from the comparison principle [2], we get $v(t) \leq w(t) \leq \frac{h_m}{1-e^{-\gamma\tau}}$. Hence the state v is bounded. Since function q verifies $1 - \xi \leq q(v) \leq 1$, then by the comparison principle and using the same approach as in [15], the subsystem defined by the $s(t, a)$, $y(t)$ and $z(t)$ variables has a unique nonnegative and bounded solution.

Since the state variables of system (3) are bounded, the objective functional $\mathcal{J}(.)$ is finite. So it is possible to define $d = \sup_{h\in\mathcal{M}} \mathcal{J}(h)$ and thus, there is a maximising sequence $(h^j)_{j\in\mathbb{N}}$ so that the sequence $(\mathcal{J}(h^j))_{j\in\mathbb{N}}$ converges to d. Since the set $\mathcal{M}$ is compact, then there exists a sub-sequence still denoted by $(h^j)_{j\in\mathbb{N}}$ that converges to $h^\star = (h_n^\star)_{n\in\{0,1,2,\dots N_f\}}$ so that $h^\star$ belongs to $\mathcal{M}$. As the objective functional $\mathcal{J}(.)$ is continuous, then it follows that $\mathcal{J}(h^\star) = d$. This achieves the proof. □

1.2 Proof of Theorem 2

Let $Q := [0, t_f] \times [0, a_\dagger]$. For any given $\bar{h} \in \mathcal{V}_h$, there is $\epsilon > 0$ small enough, such that $h^\epsilon := h + \epsilon\bar{h} \in \mathcal{M}$. Let $s^\epsilon(t, a)$, $y^\epsilon(t)$, $z^\epsilon(t)$, and $v^\epsilon(t)$ be the solution of system (3) corresponding to the control strategy h^ϵ. Then, the directional derivative of objective functional $\mathcal{J}(.)$ is given by

$$D_{\bar{h}}\mathcal{J}(h) = \lim_{\epsilon\to0^+} \epsilon^{-1}[\mathcal{J}(h + \epsilon\bar{h}) - \mathcal{J}(h)] = \int_0^{a_\dagger} \Theta(a)\lambda_s(t_f, a)da - \sum_{n=0}^{N_f} C'(h_n)\bar{h}_n - C_y\lambda_y(t_f). \quad (15)$$

Multiplying the equations of the sensitivity system (6) by the adjoint state variables $\Phi_s(t, a)$, $\Phi_y(t)$, $\Phi_z(t)$ and $\Phi_v(t)$ respectively, and also the equations (9) by the sensitivities $\lambda_v(t, a)$, $\lambda_y(t)$, $\lambda_z(t)$ and $\lambda_v(t)$ respectively. We derive the following relation:

$$\begin{aligned} \int_Q \lambda_s [\partial_t\Phi_s + \partial_a\Phi_s] \, dadt \quad &+ \int_0^{t_f} \lambda_y\dot{\Phi}_y dt + \int_0^{t_f} \lambda_z\dot{\Phi}_z dt + \int_0^{t_f} \lambda_v\dot{\Phi}_v dt \\ &+ \int_Q \Phi_s [\partial_t\lambda_s + \partial_a\lambda_s] \, dadt + \int_0^{t_f} \Phi_y\dot{\lambda}_y dt + \int_0^{t_f} \Phi_z\dot{\lambda}_z dt \\ &+ \int_0^{t_f} \Phi_v\dot{\lambda}_v dt = 0. \end{aligned} \quad (16)$$

We have the following relation:

$$
\begin{aligned}
\int_0^{t_f} \Phi_v \dot{\lambda}_v dt &= \sum_{n=0}^{N_f-1} \int_{t_n}^{t_{n+1}} \Phi_v \dot{\lambda}_v dt + \int_{t_{N_f}}^{t_f} \Phi_v \dot{\lambda}_v dt \\
&= \sum_{n=0}^{N_f-1} [\Phi_v \lambda_v]_{t_n^+}^{t_{n+1}} - \sum_{n=0}^{N_f-1} \int_{t_n}^{t_{n+1}} \dot{\Phi}_v \lambda_v dt + [\Phi_v \lambda_v]_{t_{N_f}^+}^{t_f} - \int_{t_{N_f}}^{t_f} \dot{\Phi}_v \lambda_v dt \\
&= \sum_{n=0}^{N_f-1} \Phi_v(t_{n+1}) \lambda_v(t_{n+1}) - \sum_{n=0}^{N_f} \Phi_v(t_n^+) \lambda_v(t_n^+) - \int_0^{t_f} \dot{\Phi}_v \lambda_v dt \qquad (\text{since } \Phi_v(t_f) = 0) \\
&= \sum_{n=0}^{N_f-1} \Phi_v(t_{n+1}) \lambda_v(t_{n+1}) - \sum_{n=0}^{N_f} \Phi_v(t_n^+) [\lambda_v(t_n) + \bar{h}_n] - \int_0^{t_f} \dot{\Phi}_v \lambda_v dt \\
&= -\sum_{n=0}^{N_f} \Phi_v(t_n^+) \bar{h}_n - \int_0^{t_f} \dot{\Phi}_v \lambda_v dt \qquad (\text{since } \Phi_v(t_n) = \Phi_v(t_n^+) \text{ and } \Phi_v(t_0) = 0). \qquad (17)
\end{aligned}
$$

Integrating the equation (16) and taking into account the initial and boundary conditions of equations (6)–(9) and also the relation (17), we get the relation:

$$
\int_0^{a_\dagger} \Theta(a) \lambda_s(t_f, a) da - C_y \lambda_y(t_f) - \sum_{n=0}^{N_f} \Phi_v(t_n^+) \bar{h}_n = 0. \qquad (18)
$$

Therefore,

$$
\int_0^{a_\dagger} \Theta(a) \lambda_s(t_f, a) da - \sum_{n=0}^{N_f} C'(h_n) \bar{h}_n - C_y \lambda_y(t_f) = -\sum_{n=0}^{N_f} C'(h_n) \bar{h}_n + \sum_{n=0}^{N_f} \Phi_v(t_n^+) \bar{h}_n. \qquad (19)
$$

Hence $D_{\bar{h}} \mathcal{J}(h) = \sum_{n=0}^{N_f} \left[\Phi_v(t_n^+) - C'(h_n)\right] \bar{h}_n$ for all $\bar{h} \in \mathcal{V}_h$. This achieves the proof. ☐

1.3 Proof of Theorem 3

Let $h^\star = (h_n^\star)_{n \in \{0,1,2,\ldots,N_f\}} \in \mathcal{M}$ be the optimal impulsive control which maximises the objective functional $\mathcal{J}(.)$ and let $s^\star(t,a)$, $y^\star(t)$, $z^\star(t)$ and $v^\star(t)$ be the corresponding state variables. Then, for an arbitrary but fixed $\bar{h} \in \mathcal{V}_{h^\star}$ and there is $\epsilon > 0$ small enough such that $\mathcal{J}(h^\star + \epsilon \bar{h}) \leq \mathcal{J}(h^\star)$. It is derived from Theorem 2 that $D_{\bar{h}} \mathcal{J}(h^\star) = \sum_{n=0}^{N_f} \left[\Phi_{v^\star}(t_n^+) - 2\eta h_n^\star\right] \bar{h}_n \leq 0$. Since $\bar{h}_n$ is arbitrary for $n \in \{0,1,2,\ldots,N_f\}$, which implies that $h_n^\star = \frac{\Phi_{v^\star}(t_n^+)}{2\eta}$. By taking the lower and upper bounds into account, we obtain the optimal impulsive control (12). This achieves the proof. ☐

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Symbol	Description	Value
t_f	Duration of a cropping season	250 days
$a_\dagger$	maximum age of coffee berry	250 days
g	Production rate of new coffee berries	1200 berries.day^{-1}
μ	Natural mortality rate of healthy coffee berries	0.002 day^{-1}
ε	Colonising CBB per berry (scaling factor)	1 female.berry^{-1}
$\beta(a)$	Infestation function (13):	day^{-1}
$\beta_{\min}$	minimum infestation rate	0.004 day^{-1}
β_a	age-dependent extra infestation rate	0.036 day^{-1}
k_β	infestation coefficient	0.035 day^{-1}
a_β	infestation threshold age	90 days
α	CBB–berry interaction constant	0.7 female.berry^{-1}
ϕ	Emergence rate of new colonising females	2 day^{-1}
μ_y	Natural mortality rate of colonising females	1/20 day^{-1}
μ_z	Natural mortality rate of infesting females	1/27 day^{-1}
ξ	Bio-insecticide load maximal effectiveness	0.8
k	Bio-insecticide load half saturation constant	200 g.day^{-1}
γ	Bio-insecticide decay rate	1/50 day^{-1}
η	Bio-insecticide cost	0.002 \$.g^{-2}
τ	Bio-insecticide application period	7 *or* 14 days
$h_{\max}$	Maximal bio-insecticide application	180 g
$\Theta(a)$	Coffee berry price function (14):	\$.berry^{-1}
$\overline{\Theta}$	coffee berry asymptotic price	0.025 \$.berry^{-1}
a_Θ	berry age at half asymptotic price	120 days
m	Hill constant	7 –
C_y	Cost of remaining colonising females	10^{-4} \$.female^{-1}

Table 1: Model and control parameter values. Most parameter values are based on biological data collected in the literature. See [12, 14, 15] for more details.