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OBSERVER DESIGN FOR A FISH POPULATION MODEL

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RÉSUMÉ. Le but de ce travail est d'appliquer des outils de contrôle aux systèmes de population de pêche. on construit un observateur pour un modèle continu structuré en age de population de pêche exploitée qui tient compte des pré-recrutés. Les variables du modèle: l'effort de pêche, les classes d'age et la capture sont considérés respectivement comme contrôleur, états du systèmes et sa sortie mesurée. Le changement de variables basé sur les dérivés de Lie nous a permis de mettre le système sous une forme canonique observable. La forme explicite de l'observateur est finalement donnée.

ABSTRACT. Our aim is to apply some tools of control to fishing population systems. In this paper we construct a non linear observer for the continuous stage structured model of an exploited fish population, using the fishing effort as a control term, the age classes as a states and the quantity of captured fish as a measured output. Under some biological satisfied assumptions we formulate the observer corresponding to this system and show its exponential convergence. With the Lie derivative transformation, we show that the model can be transformed to a canonical observable form; then we give the explicit gain of the estimation.

MOTS-CLÉS : Modèle Structuré, Pêche, Observateur, Population Dynamique, Ecosystème.

KEYWORDS : Structured Model, Fish, Observer, Population Dynamics, ecosystem.

1. Introduction

When solving control engineering problems, it is often necessary to know the state of a dynamical system. Most of the modern control design methods, especially for nonlinear systems, use a state feedback as the controller. Knowing the system state is also important for surveillance of a technical system, either by a human or automatically. But in most applications, it is very difficult or even impossible to measure the entire state of the system : either because applying sensors for all states would require too much effort, or because there are no methods to measure a state variable in realtime. Thus the problem of observer design is how to get an estimate for the state of a dynamical system from the knowledge of its input and output signals.This is the case of many widely diffused process control strategies. Therefore, the presence of unknown states becomes a difficulty which can be solved by means of the inclusion of an appropriate state estimator. For this reason, many researchers have focused their attention on the development of suitable algorithms to perform the estimation. In this sense, several techniques have been introduced to estimate state variables from the available measurements, usually related to meaningful variables. From the obtainable information about the process, there exist many possible kinds of estimators to be used depending on the mathematical structure of the process model. With the development of Kalman Filter and Luenberger observer [5], the linear estimation problem in the presence of white noise is almost solved. The first important results for nonlinear systems were obtained by Hermann and Krener [17], who gave a sufficient condition for local observability. Gauthier and Bornard [7] found a class of systems which are observable for any input signal. Their result is quite important for control applications, where the input is directly computed by the controller, usually without regarding whether the system is observable with this input or not. A different approach for controlled systems can be found in the work of Zeitz [14], where also derivatives of the input signal are taken into account. The Kalman-like nonlinear observer produces good estimates in the sense of mean square errors. For the system with white noises, a Kalman-like nonlinear observer is widely accepted. The high gain observer is an appropriate technique for several class of nonlinear systems. Its origin can be traced back to Gauthier [8]. The basic idea of this approach is to dominate the nonlinear behavior of the system by applying high gains to a slightly modified Luenberger observer. Convergence is then usually proven by giving a quadratic Lyapunov function, as normally used for linear systems. An extension of this observer synthesis to the multi out-put case is given in [11][12][13].
In fishery systems the states variables can't be measured, and the resources cannot be counted directly except with acoustic method which is not generalized yet. This difficulty leds some authors to estimate the biomass through available data. Ouahbi [1] construct an observer that gives an estimate of the state of the discrete time model and which is independent of the choice of stock recruitment function. J.L Gouze et al [6]. present a technique for the dynamic estimation of bounds and no-measured variables of an uncertain dynamical systems. They show the applicability of this method to the three stages structured population model ; one disadvantage of this method it is formulated under the assumption that only the oldest stage is subject to be captured. In this paper, we deal with the continuous age-structured model of population dynamics of exploited fish. The model is structured in n age classes and we assume that at each time we can measure the quantity of captured fishes and all age classes are subject to be captured. We show that it is possible to construct an observer which gives an estimation of the biomass of fishes by age class. The high gain observer technique is used based on the work of [8][13]. The exponential

observer is explicitly formulated in an invariant domain.
The paper is organized as follows. We first consider the description of the continuous stage structured model, under some biological satisfied assumptions. Next we give a state transformation in order to transform our system in a canonical observable form relying on the Lie derivative transformation. Then we investigate the technique for the estimation of the biomass in an invariant domain. In section 4 a numerical example is given and simulation results are shown.

2. Problem Formulation and Assumptions

We consider a population of exploited fish which is structured in n age classes $(n \geq 2)$, where every stage i is described by the evolution of it's biomass X_i for $0 \leq i \leq n$. Under some assumptions on the population; we can represent it's dynamics by the following system of differential equation [18] [19].

$$\left\{ \begin{array}{lcc} \dot{X_0} & = & -\alpha_0 X_0 + \sum_{i=1}^{n} f_i l_i X_i - \sum_{i=0}^{n} p_i X_i X_0 \\ \dot{X_1} & = & \alpha X_0 - (\alpha_1 + q_1 E) X_1 \\ \vdots & & \vdots \\ \dot{X_n} & = & \alpha X_{n-1} - (\alpha_n + q_n E) X_n \\ Y & = & q_1 X_1 + q_2 X_2 + ... + q_n X_n \end{array} \right. \qquad [1]$$

Where :
$\alpha_i = \alpha + M_i$.
M_i :is the natural mortality of the individuals of the i^{th} age class;
α :is the linear aging coefficient;
p_0 : is the juvenile competition parameter;
p_i : predation parameter of class i on class 0;
f_i :is the fecundity rate of class i;
l_i : is the reproduction efficiency of class i;
q_i : is the catchability of the individuals of the i^{th} age class;
X_i : is the biomass of class i;
E :is the fishing effort at time t and is regarded as an input;
Y :is the total catch per unit of effort and is regarded as output;

Let us note that all the parameters of the model are positive. The recruitment from one class to another can be represented by a strictly positive coefficient of passage. The passage rate α from the juvenile class to the adult stages is supposed to be constant with respect to time and stages. This means that the time of residence is equal to $\frac{1}{\alpha}$. The laying eggs is considered continuous with respect to time. The total number of eggs introduced in the juvenile stage is given by $\sum_1^i f_i l_i X_i$. The cannibalism term $\sum_1^i p_i X_0 X_i$ is based on the Lotka-Volterra predating term between class i and class 0. The intra-stage competition for food and space is expressed as $p_0 X_0^2$. The mortality of each stage i is caused by the fishing and natural mortality which is supposed linear [19].

One supposes that the system [1] satisfies the following assumptions :

Assumption 1 :
One non linearity at least must be considered.

$$\sum_{i=0}^{n} p_i \neq 0$$

Assumption 2 :
The spawning coefficient must be big enough so as to avoid extinction.

$$\sum_{i=1}^{n} f_i l_i \pi_i > \alpha_0$$

where : $\pi_i = \frac{\alpha^i}{\prod_{j=1}^{i}(\alpha_j + q_j \bar{E})}$ and $\bar{E}$ is a constant fishing effort.
Under the assumptions [1] and [2] the system [1] has two equilibrium points [19] :
The first one is the origin $X = 0$ which corresponds to an extincted population and is therefore not very interesting. The second one is the nontrivial equilibrium X^* defined as :

$$X_i^* = \pi_i X_0^* \text{ and } X_0^* = \frac{\sum_1^n f_i l_i \pi_i - \alpha_0}{p_0 + \sum_1^n p_i \pi_i}$$

Assumption 3 :
All age classes are subject to catch and the oldest one yields eggs.

$$\forall\ i = 1 \dots n \quad q_i > 0 \text{ and } f_n l_n \neq 0$$

Assumption 4 :
Each predator lays more eggs than it consumes.

$$X_0^* < \mu = \min_{i=1 \dots n}\left(\frac{f_i l_i}{p_i}\right) \text{ for } f_i l_i p_i \neq 0.$$

Assumption 5 :
We assume that the fishing effort is subject to the constraint.

$$0 < E_{min} \leq E \leq E_{max}$$

In [2] the authors show that the system [1] controlled by any positive constant feedback law $\overline{E}$ is asymptotically stable. They also construct a nonlinear state feedback law that allows to stabilize the system around the nontrivial steady state X^*.

3. Nonlinear Observer Design

State observers (software sensors) are able to provide a continuous estimation of some signals which are not measured by hardware sensors. They need a mathematical model of the process and hardware measurements of some other signals. An observer is a dynamic system whose input includes the control u and the output y and whose output is an estimate of the state vector $\widehat{x}$.

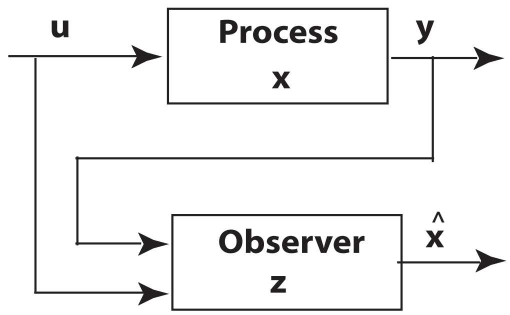

Figure 1. *Principal of the observer*

In general the biomass $X(t)$ can't be measured directly, so the problem addressed here is how to use the measurable information the input u and the output Y in order to construct an exponential observer that is to say, an auxiliary system of differential equation whose state $\widehat{X}(t)$ gives an estimate of the state $X(t)$ of the system [1]. More precisely we shall have $\lim_{t\longrightarrow+\infty}(\widehat{X}(t)-X(t))=0$ with an exponential rate of convergence.
The system [1] can be written in the following general form.

$$\left\{\begin{array}{lcl}\dot{X} & = & A_1X+B_1Xu+F(X)\\ Y & = & C_1X\end{array}\right. \qquad [2]$$

where :

$$X=(X_0,X_1,...,X_n)^\top \qquad u=E$$

$$A_1=\begin{bmatrix}0&0&0&\dots&0\\ \alpha&0&0&\dots&0\\ 0&\alpha&0&\dots&0\\ 0&0&\ddots&0&0\\ 0&0&0&\alpha&0\end{bmatrix} \qquad B_1=\begin{bmatrix}0&0&0&\dots&0\\ 0&-q_1&0&\dots&0\\ 0&0&-q_2&\dots&0\\ 0&0&\ddots&\ddots&0\\ 0&0&0&0&-q_n,\end{bmatrix}$$

$$F(X)=\begin{bmatrix}-\alpha_0X_0+\sum_{i=1}^n f_il_iX_i-\sum_{i=0}^n p_iX_iX_0\\ -\alpha_1X_1\\ \vdots\\ -\alpha_nX_n\end{bmatrix}$$
$$\text{and}\quad C_1 \;=\; [0,q_1,q_2,\dots,q_n]$$

In order to get asymptotic results. We restrict our study to the set D defined as follows $D=\Pi_1^n[a_i,b_i]$ where a_i can be chosen as small as one need and $b_i=(1+\upsilon_i)\pi_i$ with $\upsilon_0=0<\upsilon_1<...<\upsilon_n<1$. it is shown in [18] that a_i and b_i are bounded by some function of the parameter f_i,l_i and π_i and that D is an invariant domain by system [1].
F is liptschitz in D
Let us define the liptschitz constant of F
By the mean value theorem there exist a point z on the line segment joining $X^1\in D$ and $X^2\in D$ such that :

$$F(X^1) - F(X^2) = \frac{\partial F}{\partial X}(z)(X^1 - X^2)$$

Thus

$$\begin{array}{rcl} \|F(X^1) - F(X^2)\| & = & \|\frac{\partial F}{\partial X}(z)(X^1 - X^2)\| \\ & \leq & \|\frac{\partial F}{\partial X}(z)\|\|(X^1 - X^2)\| \end{array}$$

Taking into account $\quad a_i \leq X_i \leq b_i \quad \forall i \geq 0$ it follows :

$$\|\frac{\partial F}{\partial X}(z)\| \leq 2p_0b_0 + \sum_1^n p_ib_i + \sum_1^n f_il_i + (\alpha_0^2 + \alpha_1^2 \ldots + \alpha_n^2)^{\frac{1}{2}}$$

Consequently F is liptschitz in the domain D with the liptschitz constant :

$$L = 2p_0b_0 + \sum_1^n p_ib_i + \sum_1^n f_il_i + (\alpha_0^2 + \alpha_1^2 \ldots + \alpha_n^2)^{\frac{1}{2}}.$$

3.1. State Transformation

To facilitate the design of the nonlinear observer, one considers the following change of coordinates :

$$\phi : X \longrightarrow Z = (h(X), L_fh(X), \ldots, L_f^nh(X))$$

where :

$$f(X) = A_1X, h(X) = C_1X$$

And L denotes the Lie derivative operator defined as :

$$L_fh(X) = \frac{\partial h(X)}{\partial X}f(X) \text{ and } L_f^nh(X) = L_f(L_f^{n-1}h(X))$$

Z can be expressed as :

$$\begin{array}{rcl} Z & = & (C_1X, C_1A_1X, ..., C_1A_1^nX) \\ & = & MX \end{array}$$

Where :

$$M = \begin{bmatrix} 0 & q_1 & q_2 & \ldots & q_n \\ q_1\alpha & q_2\alpha & \ldots & q_n\alpha & 0 \\ q_2\alpha^2 & \ldots & q_n\alpha^2 & 0 & 0 \\ \vdots & . & 0 & 0 & 0 \\ q_n\alpha^n & 0 & 0 & 0 & 0 \end{bmatrix}$$

It is easy to see that

$$\det M = q_n^{n+1}\alpha^{\frac{n(n+1)}{2}}$$

Thus :

$$\forall \ q_n \neq 0 \quad \text{we have} \quad \det M \neq 0$$

($q_n \neq 0$ means that the last stage-class is subject to catch.)
Consequently ϕ is a diffeomorphism in D
Having recourse to some global results found out by Gauthier et al [7] and Farza et al [13] ϕ transform [2] to :

$$\left\{ \begin{array}{lcl} \dot{Z} & = & AZ + \psi(Z)u + \varphi(Z) + \omega(Z) \\ Y & = & CZ \end{array} \right. \qquad [3]$$

Where :

$$A = \begin{bmatrix} 0 & 1 & \dots & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad C = [1, 0, 0, \dots, 0]$$

$$\varphi(Z) = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ L_f^{n+1} h(\phi^{-1}(Z)) \end{bmatrix} = 0 \qquad (L_f^{n+1} h(\phi^{-1}(Z)) = C_1 A_1^{n+1} X = 0)$$

$$\psi(Z) = \begin{bmatrix} L_g L_f^0 h(\phi^{-1}(Z)) \\ L_g L_f^1 h(\phi^{-1}(Z)) \\ \vdots \\ L_g L_f^n h(\phi^{-1}(Z)) \end{bmatrix} = M B_1 M^{-1} Z \quad \text{where } g(X) = B_1 X$$

$$\omega(Z) = MF(M^{-1}Z)$$

The system [3] can be written as :

$$\left\{ \begin{array}{lcl} \dot{Z} & = & AZ + MB_1 M^{-1} Z u + MF(M^{-1}Z) \\ Y & = & CZ \end{array} \right. \qquad [4]$$

3.2. Proposition

For any initial condition $X(0) \in D$ and any $\widehat{X}(0) \in D$ and for θ large enough the system [2] satisfying assumptions [1], [2], [3], [4] and [5]. can be exponentially estimated by the following dynamical system :

$$\dot{\widehat{X}} = A_1 \widehat{X} + B_1 \widehat{X} u + F(\widehat{X}) - \theta M^{-1} d_\theta^{-1} S_1^{-1} C'(C_1 \widehat{X} - Y) \qquad . \qquad [5]$$

Where :
S_1 is the symmetric positive definite solution of the algebraic equation :

$$\theta S_\theta + A' S_\theta + S_\theta A - C'C = 0$$

For $\theta = 1$ and it can be expressed as :

$$S_1(i,j) = (-1)^{i+j} C_{i+j-2}^{j-1} \quad for \quad 1 \leq i,j \leq n \quad Where \quad C_j^i = \frac{j!}{i!(j-i)!}$$

d_θ is a diagonal matrix defined by : $d_\theta = diag(1, \frac{1}{\theta}, \ldots, \frac{1}{\theta^n})$

3.3. Lemma

For θ large enough the system below is an exponential observer for the system [4],

$$\dot{\widehat{Z}} = A\widehat{Z} + MB_1M^{-1}\widehat{Z}u + MF(M^{-1}\widehat{Z}) - \theta d_\theta^{-1} S_1^{-1} C'(C\widehat{Z} - Y) \qquad [6]$$

Proof

Let $e = \widehat{Z} - Z$
Then on can check that :

$$\dot{e} = (A - \theta d_\theta^{-1} S_1^{-1} C'C)e + MBM^{-1}ue + \Delta(F)$$

Where $\Delta(F) = MF(M^{-1}\widehat{Z}) - MF(M^{-1}Z)$
F is lipschitz with the constant L so :

$$\|\Delta(F)\| \leq L_1 \|e\|$$

Where

$$L_1 = L\|M\|\|M^{-1}\|$$

Let V_θ a candidate lyapunov equation for the system [4].

$$V_\theta = \frac{1}{\theta} e' d_\theta S_1 d_\theta e$$

The time derivative of V_θ computed along solutions of the differential equations [4] is given by :

$$\begin{aligned} \dot{V}_\theta = & \frac{1}{\theta} e'(A' d_\theta S_1 - \theta C'C) d_\theta e + 2\frac{1}{\theta} e' d_\theta S_1 d_\theta MBM^{-1}ue + e' d_\theta (S_1 d_\theta A - \theta C'C) e \\ & + 2\frac{1}{\theta}(\Delta(F)) d_\theta S_1 d_\theta e) \\ & = \frac{1}{\theta} e' d_\theta (d_\theta^{-1} A' d_\theta S_1 + S_1 d_\theta A d_\theta^{-1} - \theta d_\theta^{-1} C'C - \theta C'C d_\theta^{-1}) d_\theta e \\ & + 2\frac{1}{\theta} e' d_\theta S_1 d_\theta MBM^{-1}ue + 2\frac{1}{\theta}(\Delta(F)) d_\theta S_1 d_\theta e \end{aligned} \qquad [7]$$

Taking into account the algebraic equation :

$$\theta S_\theta + A' S_\theta + S_\theta A - C'C = 0$$

And

$$d_\theta A d_\theta^{-1} = \theta A, C'C d_\theta^{-1} = C'C$$

It follows

$$\dot{V}_\theta = \frac{1}{\theta} e' d_\theta(-\theta S_1 - \theta C'C) d_\theta e + 2\frac{1}{\theta} e' d_\theta S_1 d_\theta M B_1 M^{-1} u d_\theta^{-1} d_\theta e + 2\frac{1}{\theta}(\Delta(F)) d_\theta S_1 d_\theta e$$

We indeed get :

$$\dot{V}_\theta \leq -\theta V_\theta + \frac{2}{\theta}\lambda_{max}(S_1)(\|MB_1M^{-1}\|E_{max} + L_1)\|d_\theta e\|^2$$

Using the above inequality :

$$\lambda_{min}(S_1)\|d_\theta e\|^2 \quad \leq \quad e' d_\theta S_1 d_\theta e \quad \leq \quad \lambda_{max}(S_1)\|d_\theta e\|^2$$

where : $\lambda_{min}(S_1)$ and $\lambda_{max}(S_1)$ are respectively the minimal and the maximal eigenvalues of S_1.
it Follows :

$$\dot{V}_\theta \leq -(\theta - \theta_1) V_\theta$$

Where :

$$\theta_1 = \frac{2\lambda_{max}(S_1)(\|MB_1M^{-1}\|E_{max}+L_1)}{\lambda_{min}(S_1)}$$

Consequently

$$V_\theta(t) \leq \exp[-(\theta - \theta_1)t] V_\theta(0)$$

Then

$$\|d_\theta e(t)\| \leq \sqrt{\frac{\lambda_{max}(S_1)}{\lambda_{min}(S_1)}} \exp[-(\frac{\theta-\theta_1}{2})t]\|d_\theta e(0)\|$$

From the inequality :

$$\frac{1}{\theta^{n+1}}\|e(t)\| \quad \leq \quad \|d_\theta e(t)\| \quad \leq \quad \|e(t)\|$$

We get :

$$\|e(t)\| \leq \theta^{n+1}\sqrt{\frac{\lambda_{max}(S_1)}{\lambda_{min}(S_1)}} \exp[-(\frac{\theta - \theta_1}{2})t]\|e(0)\| \qquad [8]$$

Consequently For θ large enough $\|e\|$ converge exponentially to zero.

Proof of the Proposition
we have

$$\widehat{X} = M^{-1}\widehat{Z}$$

Then

$$\begin{aligned}
\dot{\widehat{X}} &= M^{-1}\dot{\widehat{Z}} \\
&= M^{-1}(A\widehat{Z} + MB_1M^{-1}\widehat{Z}u + MF(M^{-1}\widehat{Z}) - \theta d_\theta^{-1}S_1^{-1}C'(C\widehat{Z} - Y)) \\
&= M^{-1}A\widehat{Z} + B_1M^{-1}\widehat{Z}u + F(M^{-1}\widehat{Z}) - \theta M^{-1}d_\theta^{-1}S_1^{-1}C'(C\widehat{Z} - Y) \\
&= M^{-1}AM\widehat{X} + B\widehat{X}u + F(\widehat{X}) - \theta M^{-1}d_\theta^{-1}S_1^{-1}C'(C_1\widehat{X} - Y) \\
&= A_1\widehat{X} + B_1\widehat{X}u + F(\widehat{X}) - \theta M^{-1}d_\theta^{-1}S_1^{-1}C'(C_1\widehat{X} - Y)
\end{aligned}$$

Which end the proof of the proposition.

3.4. An observer for n=6

Below, we expose a system that we claim to be an observer for [1]. For simplicity of exposition, and to show that the observer implementation is simple and it requires small computational effort, the construction is made in the 7-dimensional case (7 age classes).

$$\left\{\begin{array}{lll}
\dot{X}_0 &=& -\alpha_0 X_0 + \sum_{i=1}^{n} f_i l_i X_i - \sum_{i=0}^{6} p_i X_i X_0 \\
\dot{X}_1 &=& \alpha X_0 - (\alpha_1 + q_1 E)X_1 \\
\dot{X}_2 &=& \alpha X_1 - (\alpha_2 + q_2 E)X_2 \\
\dot{X}_3 &=& \alpha X_2 - (\alpha_3 + q_3 E)X_3 \\
\dot{X}_4 &=& \alpha X_3 - (\alpha_4 + q_4 E)X_4 \\
\dot{X}_5 &=& \alpha X_4 - (\alpha_5 + q_5 E)X_5 \\
\dot{X}_6 &=& \alpha X_5 - (\alpha_6 + q_6 E)X_6 \\
Y &=& q_1X_1 + q_2X_2 + q_3X_3 + q_4X_4 + q_5X_5 + q_6X_6
\end{array}\right. \qquad [9]$$

The matrix of the state transformation M is expressed as :

$$M = \begin{pmatrix}
0 & q_1 & q_2 & q_3 & q_4 & q_5 & q_6 \\
q_1\alpha & q_2\alpha & q_3\alpha & q_4\alpha & q_5\alpha & q_6\alpha & 0 \\
q_2\alpha^2 & q_3\alpha^2 & q_4\alpha^2 & q_5\alpha^2 & q_6\alpha^2 & 0 & 0 \\
q_3\alpha^3 & q_4\alpha^3 & q_5\alpha^3 & q_6\alpha^3 & 0 & 0 & 0 \\
q_4\alpha^4 & q_5\alpha^4 & q_6\alpha^4 & 0 & 0 & 0 & 0 \\
q_5\alpha^5 & q_6\alpha^5 & 0 & 0 & 0 & 0 & 0 \\
q_6\alpha^6 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}.$$

$$M^{-1} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{q_6\,\alpha^6} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{q_6\,\alpha^5} & -\frac{q_5}{{q_6}^2\alpha^6} \\ 0 & 0 & 0 & 0 & \frac{1}{q_6\,\alpha^4} & -\frac{q_5}{{q_6}^2\alpha^5} & -\frac{\beta_1}{{q_6}^3\alpha^6} \\ 0 & 0 & 0 & \frac{1}{q_6\,\alpha^3} & -\frac{q_5}{{q_6}^2\alpha^4} & -\frac{\beta_1}{{q_6}^3\alpha^5} & -\frac{\beta_2}{{q_6}^4\alpha^6} \\ 0 & 0 & \frac{1}{q_6\alpha^2} & -\frac{q_5}{q_6\,\alpha^3} & -\frac{\beta_1}{{q_6}^3\alpha^4} & -\frac{\beta_2}{{q_6}^4\alpha^5} & -\frac{\beta_3}{{q_6}^5\alpha^6} \\ 0 & \frac{1}{q_6\,\alpha} & -\frac{q_5}{\alpha^2{q_6}^2} & -\frac{\beta_1}{\alpha^3{q_6}^3} & -\frac{\beta_2}{\alpha^4{q_6}^4} & -\frac{\beta_3}{\alpha^5{q_6}^5} & -\frac{\beta_4}{{q_6}^6\alpha^6} \\ \frac{1}{q_6} & -\frac{q_5}{\alpha{q_6}^2} & -\frac{\beta_1}{\alpha^2{q_6}^3} & -\frac{\beta_2}{{q_6}^4\alpha^3} & -\frac{\beta_3}{\alpha^4{q_6}^5} & -\frac{\beta_4}{{q_6}^6\alpha^5} & +\frac{\beta_5}{\alpha^6{q_6}^6} \end{pmatrix}.$$

Where

$$\begin{aligned}
\beta_1 &= q_4\,q_6 - {q_5}^2;\\
\beta_2 &= -2\,q_5\,q_6\,q_4 + {q_5}^3 + q_3\,{q_6}^2;\\
\beta_3 &= {q_6}^3q_2 - {q_6}^2{q_4}^2 - 2\,q_3\,{q_6}^2q_5 + 3\,{q_5}^2q_6\,q_4 - {q_5}^4;\\
\beta_4 &= {q_6}^4q_1 + 3\,q_5\,{q_6}^2{q_4}^2 - 2\,q_4\,{q_6}^3q_3 - 4\,{q_5}^3q_6\,q_4 - 2\,q_2\,q_5\,{q_6}^3 + 3\,{q_6}^2{q_5}^2q_3 + {q_5}^5;\\
\beta_5 &= 2\,{q_6}^4q_5\,q_1 - {q_6}^3{q_4}^3 + 6\,{q_6}^2{q_5}^2{q_4}^2 + 2\,q_4\,q_2\,{q_6}^4 - 6\,q_4\,q_5\,{q_6}^3q_3 - 5\,q_4\,{q_5}^4q_6\\
&\quad + {q_6}^4{q_3}^2 - 3\,{q_5}^2q_2\,{q_6}^3 + 4\,{q_5}^3{q_6}^2q_3 + {q_5}^6;
\end{aligned}$$

The other matrix that appear in the gain of estimation $-\theta M^{-1} d_\theta^{-1} S_1^{-1} C'$ are given by :

$$d_\theta = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\theta} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\theta^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\theta^3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\theta^4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\theta^5} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\theta^6} \end{pmatrix}. \qquad S_1^{-1}C' = \begin{pmatrix} C_1^7 \\ C_2^7 \\ C_3^7 \\ C_4^7 \\ C_5^7 \\ C_6^7 \\ C_6^7 \end{pmatrix} \quad .$$

The system [9] can be exponentially estimated by the following dynamical system :

$$\left\{\begin{array}{lcl}
\dot{\widehat{X_0}} &=& -\alpha_0\widehat{X_0} + \sum_{i=1}^{6} f_i l_i \widehat{X_i} - \sum_{i=0}^{6} p_i \widehat{X_i}\widehat{X_0} - \theta P_0(\theta)(\sum_{i=0}^{6} q_i \widehat{X}_i - Y)\\
\dot{\widehat{X_1}} &=& \alpha\widehat{X_0} - (\alpha_1 + q_1 E)\widehat{X_1} - \theta P_1(\theta)(\sum_{i=0}^{6} q_i \widehat{X}_i - Y)\\
\dot{\widehat{X_2}} &=& \alpha\widehat{X_1} - (\alpha_2 + q_2 E)\widehat{X_2} - \theta P_2(\theta)(\sum_{i=0}^{6} q_i \widehat{X}_i - Y)\\
\dot{\widehat{X_3}} &=& \alpha\widehat{X_2} - (\alpha_3 + q_3 E)\widehat{X_3} - \theta P_3(\theta)(\sum_{i=0}^{6} q_i \widehat{X}_i - Y)\\
\dot{\widehat{X_4}} &=& \alpha\widehat{X_3} - (\alpha_4 + q_4 E)\widehat{X_4} - \theta P_4(\theta)(\sum_{i=0}^{6} q_i \widehat{X}_i - Y)\\
\dot{\widehat{X_5}} &=& \alpha\widehat{X_4} - (\alpha_5 + q_5 E)\widehat{X_5} - \theta P_5(\theta)(\sum_{i=0}^{6} q_i \widehat{X}_i - Y)\\
\dot{\widehat{X_6}} &=& \alpha\widehat{X_5} - (\alpha_6 + q_6 E)\widehat{X_6} - \theta P_6(\theta)(\sum_{i=0}^{6} q_i \widehat{X}_i - Y)\\
Y &=& q_1 X_1 + q_2 X_2 + q_3 X_3 + q_4 X_4 + q_5 X_5 + q_6 X_6
\end{array}\right. \qquad [10]$$

where

$P_0(\theta) = \frac{\theta^6 C_7^7}{q_6\ \alpha^6}$;

$P_1(\theta) = \frac{\theta^5 C_6^7}{q_6\ \alpha^5} - \frac{\theta^6 C_7^7 q_5}{{q_6}^2 \alpha^6}$;

$P_2(\theta) = \frac{\theta^4 C_5^7}{q_6\ \alpha^4} - \frac{q_5\ \theta^5 C_6^7}{{q_6}^2 \alpha^5} - \frac{\beta_1 \theta^6 C_7^7}{{q_6}^3 \alpha^6}$

$P_3(\theta) = \frac{\theta^3 C_4^7}{q_6\ \alpha^3} - \frac{q_5\ \theta^4 C_5^7}{{q_6}^2 \alpha^4} - \frac{\beta_1 \theta^5 C_6^7}{{q_6}^3 \alpha^5} - \frac{\beta_2 \theta^6 C_7^7}{{q_6}^4 \alpha^6}$;

$P_4(\theta) = \frac{\theta^2 C_3^7}{q_6\ \alpha^2} - \frac{q_5\ \theta^3 C_4^7}{{q_6}^2 \alpha^3} - \frac{\beta_1 \theta^4 C_5^7}{{q_6}^3 \alpha^4} - \frac{\beta_2 \theta^5 C_6^7}{{q_6}^4 \alpha^5} - \frac{\beta_3 \theta^6 C_7^7}{{q_6}^5 \alpha^6}$;

$P_5(\theta) = \frac{\theta\, C_2^7}{q_6\ \alpha} - \frac{q_5\ \theta^2 C_3^7}{\alpha^2 {q_6}^2} - \frac{\beta_1 \theta^3 C_4^7}{\alpha^3 {q_6}^3} - \frac{\beta_2 \theta^4 C_5^7}{\alpha^4 {q_6}^4} - \frac{\beta_3 \theta^5 C_6^7}{\alpha^5 {q_6}^5} - \frac{\beta_4 \theta^6 C_7^7}{{q_6}^6 \alpha^6}$;

$P_6(\theta) = \frac{7}{q_6} - \frac{q_5\ \theta\, C_2^7}{\alpha {q_6}^2} - \frac{\beta_1 \theta^2 C_3^7}{\alpha^2 {q_6}^3} - \frac{\beta_2 \theta^3 C_4^7}{{q_6}^4 \alpha^3} - \frac{\beta_3 \theta^4 C_5^7}{\alpha^4 {q_6}^5} - \frac{\beta_4 \theta^5 C_6^7}{{q_6}^6 \alpha^5} + \frac{\beta_5 \theta^6 C_7^7}{\alpha^6 {q_6}^7})$;

4. Results and Discussion

One considers here a population with five stages age (n=4) :
Stage 0 represents the biomass of juvenile ; stage 1 represents the young adults biomass without reproduction and cannibalism ; the stages 2,3 and 4 are adults biomass with the same term of predation and the same proportion on the female mature but have different reproduction rate ($l_2 \leq l_3 \leq l_4$).

The system [1] for $n = 4$ is given by :

$$\left\{ \begin{array}{lcl} \dot{X}_0 & = & -\alpha_0 X_0 + \sum_{i=1}^{4} f_i l_i X_i - \sum_{i=0}^{4} p_i X_i X_0 \\ \dot{X}_1 & = & \alpha X_0 - (\alpha_1 + q_1 E) X_1 \\ \dot{X}_2 & = & \alpha X_1 - (\alpha_2 + q_2 E) X_2 \\ \dot{X}_3 & = & \alpha X_2 - (\alpha_3 + q_3 E) X_3 \\ \dot{X}_4 & = & \alpha X_3 - (\alpha_4 + q_4 E) X_4 \\ Y & = & q_1 X_1 + q_2 X_2 + q_3 X_3 + q_4 X_4 \end{array} \right. \qquad [11]$$

The inverse of the state transformation matrix M is expressed as :

$$M^{-1} = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{q_4 \alpha^4} \\ 0 & 0 & 0 & \frac{1}{q_4 \alpha^4} & -\frac{q_3}{q_4^2 \alpha^4} \\ 0 & 0 & \frac{1}{q_4 \alpha^2} & -\frac{q_3}{q_4^2 \alpha^3} & -\frac{\left(q_2 q_4 - {q_3}^2\right)}{q_4^3 \alpha^4} \\ 0 & \frac{1}{q_4 \alpha} & -\frac{q_3}{q_4^2 \alpha^2} & -\frac{\left(q_2\ q_4 - q_3^2\right)}{q_4^3 \alpha^3} & -\frac{\left(q_4^2 q_1 - 2 q_4 q_2 q_3 + q_3^3\right)}{q_4^4 \alpha^4} \\ \frac{1}{q_4} & -\frac{q_3}{q_4^2 \alpha^1} & -\frac{\left(q_2\ q_4 - q_3^2\right)}{q_4^3 \alpha^2} & -\frac{\left(q_4^2 q_1 - 2 q_4 q_2 q_3 + q_3^3\right)}{q_4^4 \alpha^3} & \delta_3 \end{pmatrix}$$

Where

$$\delta_3 = \frac{\left(q_3^3 q_1 - 3 q_3^2 q_4\ q_2 + q_3\ q_4^2 q_1 + q_3^4 + q_2^2\ q_4^2\right)}{q_4^5 \alpha^4}$$

The system [11] can be exponentially estimated by the following dynamical system :

$$\left\{\begin{array}{lcl}\dot{\widehat{X_0}} & = & -\alpha_0\widehat{X_0} + \sum_{i=1}^4 f_i l_i \widehat{X_i} - \sum_{i=0}^4 p_i \widehat{X_i}\widehat{X_0} - \theta Q_0(\theta)(\sum_{i=0}^4 q_i \widehat{X}_i - Y) \\ \dot{\widehat{X_1}} & = & \alpha\widehat{X_0} - (\alpha_1 + q_1 E)\widehat{X_1} - \theta Q_1(\theta)(\sum_{i=0}^4 q_i \widehat{X}_i - Y) \\ \dot{\widehat{X_2}} & = & \alpha\widehat{X_1} - (\alpha_2 + q_2 E)\widehat{X_2} - \theta Q_2(\theta)(\sum_{i=0}^4 q_i \widehat{X}_i - Y) \\ \dot{\widehat{X_3}} & = & \alpha\widehat{X_2} - (\alpha_3 + q_3 E)\widehat{X_3} - \theta Q_3(\theta)(\sum_{i=0}^4 q_i \widehat{X}_i - Y) \\ \dot{\widehat{X_4}} & = & \alpha\widehat{X_3} - (\alpha_4 + q_4 E)\widehat{X_4} - \theta Q_4(\theta)(\sum_{i=0}^4 q_i \widehat{X}_i - Y) \\ Y & = & q_1 X_1 + q_2 X_2 + q_3 X_3 + q_4 X_4 \end{array}\right. \quad [12]$$

. Where

$Q_0(\theta) = \frac{\theta^4 C_5^5}{q_4\, a^4}$

$Q_1(\theta) = \frac{\theta^3 C_4^5}{q_4\, a^3} - \frac{q_3 \theta^4 C_5^5}{q_4^2\, a^4}$

$Q_2(\theta) = \frac{\theta^2 C_3^5}{q_4\, a^2} - \frac{q_3 \theta^3 C_4^5}{q_4^2\, a^3} - \frac{\left(q_2\, q_4^2 - q_3{}^2\right)\theta^4 C_5^5}{q_4^2\, a^4}$

$Q_3(\theta) = \frac{\theta\, C_2^5}{q_4\, a} - \frac{q_3\, \theta^2 C_3^5}{q_4^2\, a^2} - \frac{\left(q_2\, q_4 - q_3^2\right)\theta^3 C_4^5}{q_4^3\, a^3} - \frac{\left(q_4^2\, q_1 - 2 q_4\, q_2\, q_3 + q_3^3\right)\theta^4 C_5^5}{q_4^4\, a^4}$

$Q_4(\theta) = \frac{5}{q_4} - \frac{q_3\, \theta\, C_2^5}{q_4^2\, a} - \frac{\left(q_2\, q_4 - q_3^2\right)\theta^2 C_3^5}{q_4^3\, a^2} - \frac{\left(q_4^2\, q_1 - 2 q_4\, q_2\, q_3 + q_3^3\right)\theta^3 C_4^5}{q_4^4\, a^3} + \delta_1 \theta^4 C_5^5$

The results obtained from the observer are illustrated by the example characterized by the parameter value inspired from literature data [18] given in table1. Here we have employed for the simulation a constant fishing effort $E(t) = \overline{E}$ and arbitrary initial states

$$X(0) = (5, 8, 10, 10, 8)$$
$$\widehat{X}(0) = (6, 4, 5, 10, 8)$$

It is shown from figure 2 and 3 that the observer converges asymptotically.
In order to show the effect of high gain, we first simulate the proposed system with the high gain $\theta = 15$ and the results are presented in figures 2 which give time evolution of the stage age X_i and theirs estimates $\widehat{X_i}$ respectively for $i = 0$ to 4 . Then in Figures 3 we give the simulation results with the high gain $\theta = 30$. Both the two values of θ guarantees asymptotic convergence, and the second one shows good tracking performances than the first.

stage i	0	1	2	3	4	stage i	0	1	2	3	4
p_i	0.2	0	0.1	0.1	0.1	M_i	0.5	0.2	0.2	0.1	0.05
f_i		0.5	0.5	0.5	0.5	α			0.8		
l_i		0	10	20	15	α_i	1.3	1	1	0.9	0.85
m_i	0.5	0.2	0.2	0.2	0.2	E			1		
q_i	0	0	0	0.1	0.15	X_{ini}	5	8	10	10	8
$\widehat{X}_{ini}$	6	4	5	10	8						

Tableau 1. *simulation data*

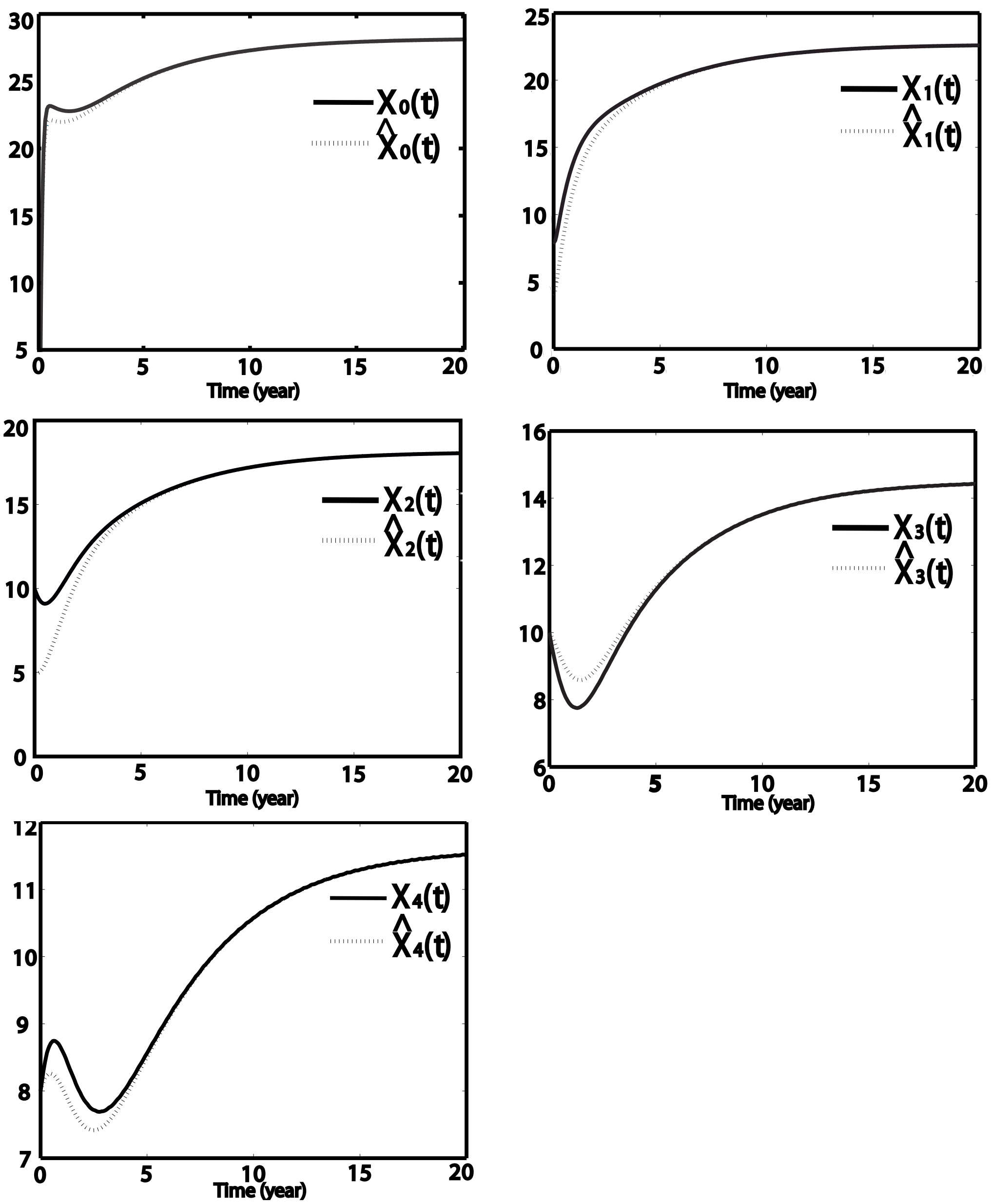

Figure 2. *Convergence asymptotic of the observer with the high gain* $\theta = 15$

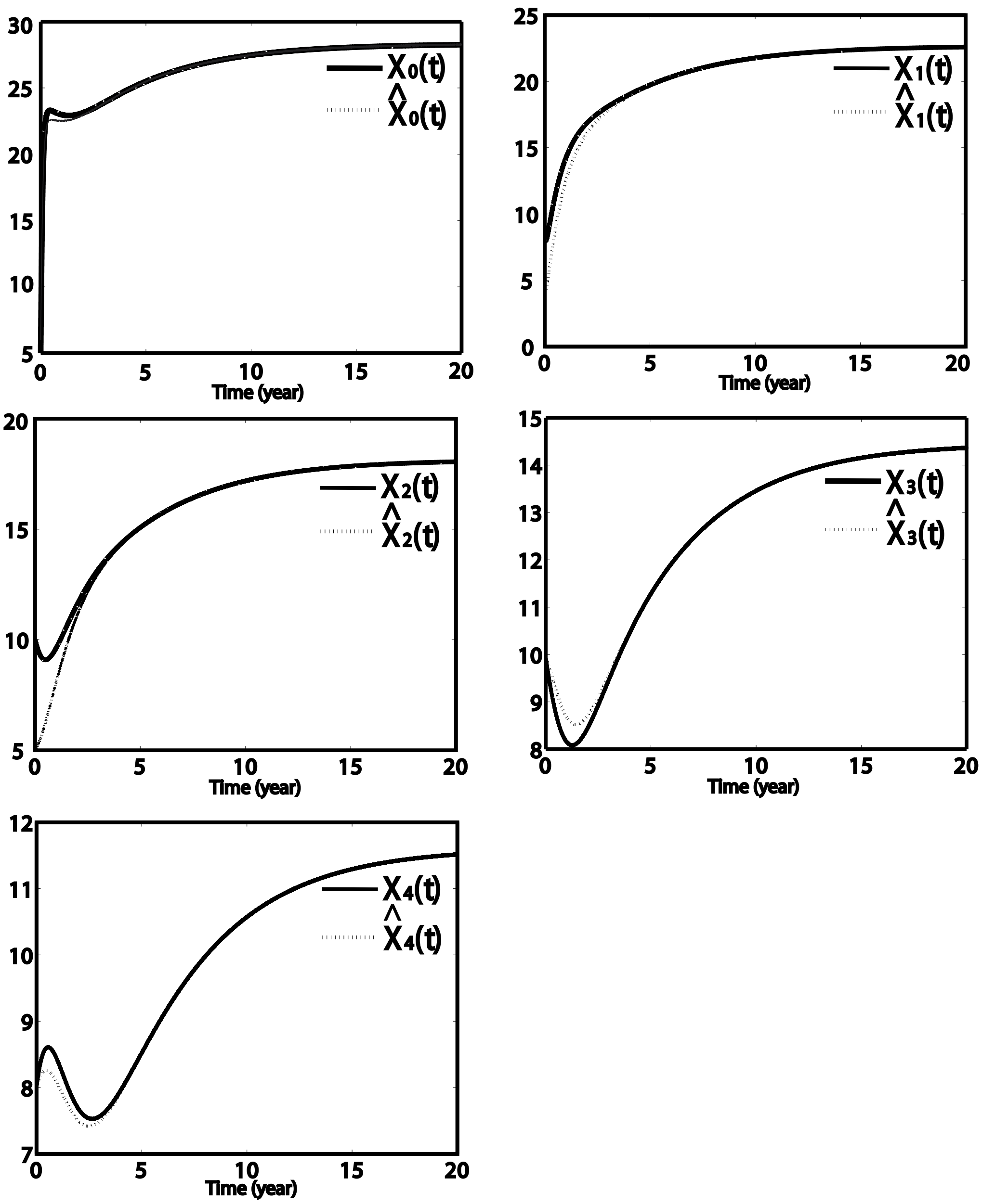

Figure 3. *Convergence asymptotic of the observer with the high gain* $\theta = 30$

5. Conclusion

We are interested in constructing a simple observer for the harvested fish population model structured in n ages classes, in an invariant domain using the Lie Derivative transformation. The high gain observer technique is used. The exponential convergence of the estimation error is proved under certain conditions and the gain of the observer is explicitly formulated. The ongoing research work focuses on the stability and the estimation of the states when stock is subjected to environmental fluctuations and disturbances.

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