

**OPTIMIZATION OF ABSORPTION SYSTEMS: CASE OF  
THE REFRIGERATORS AND HEAT PUMPS**

by

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**ABSTRACT.** The new thermo-ecological performance optimization of absorption is investigated by taking the ecological coefficient of performance ECOP as an objective function. ECOP has been expressed in terms of the temperatures of the working fluid in the main components of the system. The maximum of ECOP and the corresponding optimal temperatures of the working fluid and other optimal performance design parameters such as coefficient of performance, specific cooling load of absorption refrigerators, specific heating load of absorption heat pumps, specific entropy generation rate and the distributions of the heat exchanger areas have been derived analytically. The obtained results may provide a general theoretical tool for the ecological design of absorption refrigerators and heat pumps.

**KEYWORDS:** absorption, refrigerator, heat pump, ecological coefficient of performance.

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## 1. Introduction

Some authors have highlighted the important role of finite-time thermodynamics in modelling and optimizing energy systems and absorption cycles [1-10]. The finite-time thermodynamics optimization based on various performance criteria for absorption refrigerators and absorption heat pumps are available in the literature [11-32]. In these optimization studies, the optimal performance characteristics have been investigated for the coefficient of performance, cooling load, heating load, total heat transfer area objectives and thermo-economic performance criterion by taking into account absorption refrigerators and absorption heat pumps models operating between three or four temperature levels. In the above referenced works, the objective functions chosen for optimization of absorption refrigerators and absorption heat pumps only take into account the first law of thermodynamics. The second law of thermodynamics is taken into account by the ecological optimization criterion (E) and the ecological coefficient of performance criterion (ECOP).

For an ecological design, Qin et al. [33-35], Yan and Lin [36], Huishan [37], Chen et al. [38] and Tao et al. [39] performed the thermo-ecological optimization works for absorption systems for cooling and heating applications. Yan and Lin [36] obtained that the ecological optimization criterion (E) is more advantageous than the maximum cooling load in terms of entropy production and therefore, it is beneficial for making a more rational use of energy of absorption systems so as to save energy. However the objective function defined in these thermo-ecological optimization studies has limits. It may take negative values. Such an objective function in a performance analysis can be defined mathematically; however, it needs interpretation to comprehend this situation thermodynamically [40-42].

The new thermo-ecological performance optimization of an absorption refrigerators and heat pumps with the losses of heat resistance, internal irreversibility and leakage is investigated by taking the ECOP as an objective function. The ECOP is defined as the cooling rate or heating rate per unit loss rate of availability. The ecological coefficient of performance has been expressed in terms of the temperatures of the working fluid in the main components of the considered absorption systems which are the generator, evaporator, condenser and absorber. The maximum of the new thermo-ecological objective function and the corresponding optimal temperatures of the working fluid and other optimal performance design parameters have been derived analytically.

## 2. Model description.

The main components of an absorption refrigeration or heat pump system are a generator, an absorber, a condenser and an evaporator as shown schematically in Fig. 1. In the shown model  $\dot{Q}_H$  is the rate heat absorbed from the heat source at temperature  $T_H$  to generator,  $\dot{Q}_C$  and  $\dot{Q}_A$  are, respectively, the heat rejection rates from the condenser and absorber at temperature  $T_C$  and  $T_A$  respectively and  $\dot{Q}_L$  is the heat input rate at temperature  $T_L$  to the evaporator. The work input required by the solution pump is negligibly small compared to the energy input to the generator and therefore, is often neglected in the analysis.

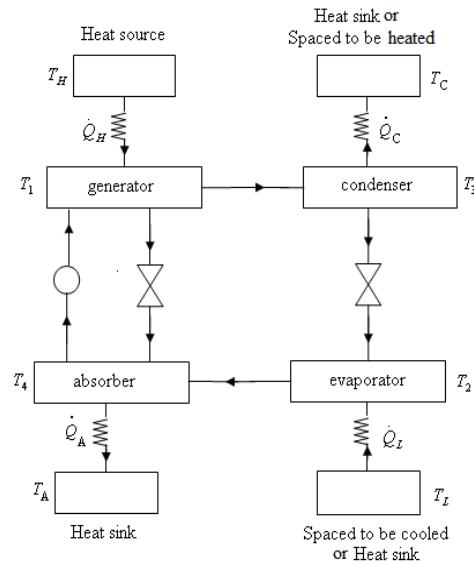


Fig. 1: Absorption system

Absorption refrigeration systems operate between three temperature levels, if  $T_A = T_C$  (three-heat-source model) or four temperature levels when  $T_A \neq T_C$  (four-heat-source model). In this work both three- and four-heat-source models, as shown in Figs. 2 and 3, are considered. The cycle of the working fluid consists of irreversible isothermal and irreversible adiabatic processes. The temperatures of the working fluid in the isothermal processes are different from those of the external heat reservoirs so that heat is transferred under a finite temperature difference, as shown in Figs. 2a and b. In Fig. 2,  $\dot{Q}_O = \dot{Q}_C + \dot{Q}_A$ .  $T_1$ ,  $T_2$ ,  $T_3$  and  $T_4$  are, respectively, the temperatures of the

working fluid in the generator, evaporator, absorber and condenser. It is assumed that the working fluid in the condenser and absorber has the same temperature  $T_3$  for three-heat-source models. The heat exchanges between the working fluid and heat reservoirs obey a linear heat transfer law.

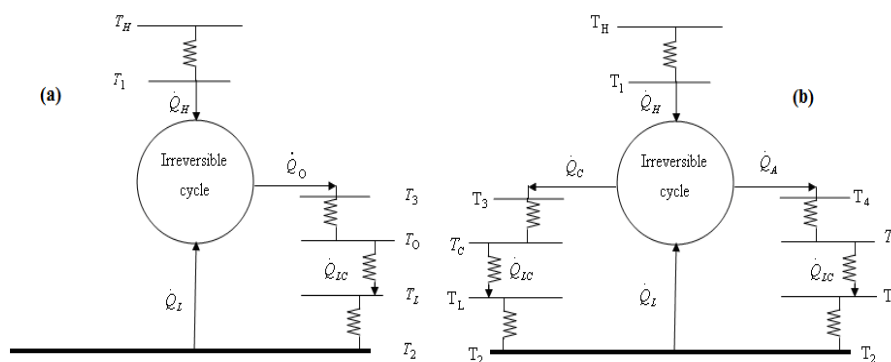


Fig. 2 : Absorption cycle models : (a) three-heat-source model and (b) four-heat-source model.

Based on standard definition of the coefficient of performance, specific cooling rate, specific heating rate and specific rate of entropy production and on the new ecological performance optimization technique applying the ecological coefficient of performance (ECOP) as objective function the followings have been obtained.

### 3. Irreversible three-heat-source absorption refrigerators and heat pumps

#### 3.1. Three-heat-source refrigerators

**Coefficient of performance**

$$COP_{ref} = \frac{T_2(T_1 - T_3)}{T_1(T_3 - T_2)} \left\{ 1 - \xi(T_o - T_L) \left[ \frac{1}{U_L(T_L - T_2)} + \frac{T_1(T_3 - T_2)}{U_H(T_H - T_1)(T_1 - T_3)T_2} + \frac{T_3(T_1 - T_2)}{U_o(T_3 - T_o)(T_1 - T_3)T_2} \right] \right\} \quad (1)$$

**The specific cooling load**

$$r = \frac{\dot{Q}_L - \dot{Q}_{LC}}{A} = \left[ \frac{1}{U_L(T_L - T_2)} + \frac{T_1(IT_3 - T_2)}{U_H(T_H - T_1)(T_1 - IT_3)T_2} + \frac{IT_3(T_1 - T_2)}{U_o(T_3 - T_o)(T_1 - IT_3)T_2} \right]^{-1} - \xi(T_o - T_L) \quad (2)$$

where  $\xi = K_{LC}/A$ ,  $A$  is the total heat-transfer area between the cycle system and the external heat reservoirs.

### The specific entropy generation rate

$$s = \left( \frac{1}{T_L} - \frac{1}{T_o} \right) \left\{ \xi(T_o - T_L) - \left[ 1 - \frac{COP_{ref,r} T_1 (IT_3 - T_2)}{T_2 (T_1 - IT_3)} \right] \times \left[ \frac{1}{U_L(T_L - T_2)} + \frac{T_1(IT_3 - T_2)}{U_H(T_H - T_1)(T_1 - IT_3)T_2} + \frac{IT_3(T_1 - T_2)}{U_o(T_3 - T_o)(T_1 - IT_3)T_2} \right]^{-1} \right\} \quad (3)$$

where  $COP_{ref,r}$  is the coefficient of performance for reversible three-heat-source refrigerators.

### Ecological coefficient of performance (ECOP)

$$ECOP_{ref} = \frac{1/\left[ T_{env} (T_o^{-1} - T_L^{-1}) \right]}{1 - \frac{COP_{ref,r}(y-1)}{1-x} \left\{ 1 - \xi(T_o - T_L) \left[ \frac{y}{U_L(T_L y - z)} + \frac{x(y-1)}{U_H(T_H x - z)(1-x)} + \frac{y-x}{U(z-T)(1-x)} \right] \right\}^{-1}} \quad (4)$$

where  $T_{env}$  is the temperature in the environment conditions,  $x = IT_3/T_1$ ,  $y = IT_3/T_2$ ,  $z = IT_3$ ,  $T = IT_o$  and  $U = U_o/I$ .

It's been demonstrated that for a given total heat-transfer area  $A$ , when the ecological coefficient of performance attains its maximum  $ECOP_{ref,max}$ ,  $x$ ,  $y$  and  $z$  are, respectively, given by

$$x = \frac{T}{T_H} B_1 (D + B) \quad y = \frac{T}{T_L} D \quad z = T \frac{D + b_2}{1 + b_2} \quad (5)$$

$$\text{Where } B = \frac{b_2 - b_1}{1 + b_1}, B_1 = \frac{1 + b_1}{1 + b_2}, b_1 = \sqrt{U/U_H}, b_2 = \sqrt{U/U_L} \quad D = \frac{1 + \sqrt{d_1 [1 - (1 - d_1) T_L/T]}}{1 - d_1}, d_1 = \xi \frac{(1 + b_2)^2}{U} \left( \frac{T_o}{T_L} - 1 \right)$$

Thus, when the ecological coefficient of performance is a maximum, the temperatures of the working fluid in the three isothermal processes are, respectively, determined by:

$$T_1^* = T_H \frac{D + b_2}{(1 + b_1)(D + B)} \quad T_2^* = T_L \frac{1 + b_2/D}{1 + b_2} \quad T_3^* = T_O \frac{D + b_2}{1 + b_2} \quad (6)$$

which obviously lead to the optimal coefficient of performance, the optimal specific cooling load and the optimal specific entropy generation rate at the maximum ECOP conditions for three-heat-source refrigerators.

**Optimum relation for the distribution of the heat-transfer areas**

$$\sqrt{U_G A_G^*} + \sqrt{U_E A_E^*} = \sqrt{U A_O^*} \quad (7)$$

**3.1. Three-heat-source heat-pumps**

**The coefficient of performance**

$$COP_{hp} = \frac{\dot{Q}_O - \dot{Q}_{LC}}{\dot{Q}_H} = \frac{\dot{Q}_O}{\dot{Q}_H} \left( 1 - \frac{\dot{Q}_{LC}}{\dot{Q}_O} \right) = \frac{IT_3(T_2 - T_1)}{T_1(T_2 - IT_3)} \times \left\{ 1 - \xi(T_O - T_L) \left[ \frac{T_1(IT_3 - T_2)}{U_H(T_H - T_1)(T_1 - T_2)IT_3} + \frac{T_2(IT_3 - T_1)}{U_L(T_L - T_2)(T_2 - T_1)IT_3} + \frac{1}{U_O(T_3 - T_O)} \right] \right\} \quad (8)$$

**The specific heating load**

$$q = \frac{\dot{Q}_O - \dot{Q}_{LC}}{A} = \left[ \frac{T_1(IT_3 - T_2)}{U_H(T_H - T_1)(T_1 - T_2)IT_3} + \frac{T_2(IT_3 - T_1)}{U_L(T_L - T_2)(T_2 - T_1)IT_3} + \frac{1}{U_O(T_3 - T_O)} \right]^{-1} - \xi(T_O - T_L) \quad (9)$$

**The specific rate of entropy-generation rates**

$$s = \left( \frac{1}{T_E} - \frac{1}{T_O} \right) \left\{ \xi(T_O - T_E) + \left[ \frac{COP_{hp,r} T_1(T_2 - IT_3)}{IT_3(T_2 - T_1)} - 1 \right] \right\}$$

$$\times \left[ \frac{T_1(IT_3 - T_2)}{U_G(T_G - T_1)(T_1 - T_2)IT_3} + \frac{T_2(IT_3 - T_1)}{U_E(T_E - T_2)(T_2 - T_1)IT_3} + \frac{1}{U_O(T_3 - T_O)} \right]^{-1} \quad (10)$$

where  $COP_{hp, r}$  is the coefficient of performance for reversible three-heat-source heat pumps.

### The ecological coefficient of performance (ECOP)

$$ECOP_{hp} = \frac{1}{-1 + \frac{COP_{hp, r}(1-y)}{x-y} \left\{ 1 - \xi(T_O - T_L) \left[ \frac{x(1-y)}{U_H(T_H x - z)(x-y)} + \frac{y(1-x)}{U_L(T_L y - z)(y-x)} + \frac{1}{U(z-T)} \right] \right\}^{-1}} \quad (11)$$

The temperature of the working fluid in the generator, evaporator, absorber and condenser which correspond to the maximum ECOP are as follows:

$$T_1^* = T_H B / [(1+b_1)B - b_1 T] \quad T_2^* = T_L B / [(1+b_2)B - b_2 T] \quad T_3^* = B/I \quad (12)$$

where

$$B = \frac{[UT_L - (T_O - T_L)(1+b_2)b_2\xi]T + \sqrt{[(T - T_L)U + (T_O - T_L)(1+b_2)^2\xi](T_O - T_L)T_L T \xi}}{UT_L - (T_O - T_L)(1+b_2)^2\xi}$$

These optimal temperatures obviously lead to the optimal coefficient of performance, the optimal specific heating load and the optimal specific entropy generation rate at the maximum ECOP conditions for three-heat-source heat pumps.

Optimum relation for the distribution of heat-transfer areas

$$\sqrt{U_G} A_G^* + \sqrt{U_E} A_E^* = \sqrt{U} A_O^* \quad (13)$$

## 4. Irreversible four-heat-source absorption refrigerators and heat pumps

### 4.1. Four-heat-source refrigerators

The coefficient of performance

$$\begin{aligned}
 COP_{ref} &= \frac{(IT_3)^{-1} + m(IT_4)^{-1} - (1+m)T_1^{-1}}{(1+m)T_2^{-1} - (IT_3)^{-1} - m(IT_4)^{-1}} \\
 &\times \left\{ 1 - C_1 \left[ \frac{(1+m)T_2^{-1} - (IT_3)^{-1} - m(IT_4)^{-1}}{U_H(T_H - T_1) [(IT_3)^{-1} + m(IT_4)^{-1} - (1+m)T_1^{-1}]} + \frac{1}{U_L(T_L - T_2)} + \frac{T_2^{-1} - T_1^{-1}}{U_C(T_3 - T_c) [(IT_3)^{-1} + m(IT_4)^{-1} - (1+m)T_1^{-1}]} \right] \right. \\
 &\left. + \frac{m(T_2^{-1} - T_1^{-1})}{U_A(T_4 - T_A) [(IT_3)^{-1} + m(IT_4)^{-1} - (1+m)T_1^{-1}]} \right\} \quad (14)
 \end{aligned}$$

**The specific cooling load**

$$\begin{aligned}
 r = \frac{\dot{Q}_L - \dot{Q}_{LC}}{A} &= -C_1 + \left\{ \frac{(1+m)T_2^{-1} - (IT_3)^{-1} - m(IT_4)^{-1}}{U_H(T_H - T_1) [(IT_3)^{-1} + m(IT_4)^{-1} - (1+m)T_1^{-1}]} + \frac{1}{U_L(T_L - T_2)} \right. \\
 &\left. + \frac{T_2^{-1} - T_1^{-1}}{U_C(T_3 - T_c) [(IT_3)^{-1} + m(IT_4)^{-1} - (1+m)T_1^{-1}]} + \frac{m(T_2^{-1} - T_1^{-1})}{U_A(T_4 - T_A) [(IT_3)^{-1} + m(IT_4)^{-1} - (1+m)T_1^{-1}]} \right\}^{-1} \quad (15)
 \end{aligned}$$

**The specific entropy generation rate**

$$\begin{aligned}
 s &= C_6 + C_7 \frac{(1+m)T_2^{-1} - (IT_3)^{-1} - m(IT_4)^{-1}}{(IT_3)^{-1} + m(IT_4)^{-1} - (1+m)T_1^{-1}} + \left[ C_8 + C_9 \frac{(1+m)T_2^{-1} - (IT_3)^{-1} - m(IT_4)^{-1}}{(IT_3)^{-1} + m(IT_4)^{-1} - (1+m)T_1^{-1}} \right] \\
 &\times \left\{ -C_1 + \left[ \frac{(1+m)T_2^{-1} - (IT_3)^{-1} - m(IT_4)^{-1}}{U_H(T_H - T_1) [(IT_3)^{-1} + m(IT_4)^{-1} - (1+m)T_1^{-1}]} + \frac{1}{U_L(T_L - T_2)} + \frac{T_2^{-1} - T_1^{-1}}{U_C(T_3 - T_c) [(IT_3)^{-1} + m(IT_4)^{-1} - (1+m)T_1^{-1}]} \right] \right. \\
 &\left. + \frac{m(T_2^{-1} - T_1^{-1})}{U_A(T_4 - T_A) [(IT_3)^{-1} + m(IT_4)^{-1} - (1+m)T_1^{-1}]} \right\} \quad (16)
 \end{aligned}$$

Where  $C_1 = \xi(T_A - T_L + T_c - T_L)$  ;  $C_2 = \xi(T_A - T_L) \left( \frac{T_c^{-1} + mT_A^{-1}}{1+m} - T_A^{-1} \right) + \xi(T_c - T_L) \left( \frac{T_c^{-1} + mT_A^{-1}}{1+m} - T_c^{-1} \right)$



$$C_3 = \xi(T_A - T_L + T_C - T_L) \left( \frac{T_C^{-1} + mT_A^{-1}}{1+m} - T_H^{-1} \right); \quad C_4 = \frac{T_C^{-1} + mT_A^{-1}}{1+m} - T_L^{-1} \quad \text{and} \quad C_5 = \frac{T_C^{-1} + mT_A^{-1}}{1+m} - T_H^{-1}$$

### The ecological coefficient of performance ECOP

$$ECOP_{ref} = \frac{r}{T_{env} S} = \frac{1}{T_{env} \left\{ C_4 + C_5 \frac{b_3 - b_1 - b_4}{b_1 + b_4 - b_2} + \left[ C_2 + C_3 \frac{b_3 - b_1 - b_4}{b_1 + b_4 - b_2} \right] r^{-1} \right\}} \quad (17)$$

$$\text{where } r = -C_1 + \left\{ \frac{b_3 - b_1 - b_4}{U_H [T_H - (1+m)/b_2] (b_1 + b_4 - b_2)} + \frac{1}{U_L [T_L - (1+m)/b_3]} \right. \\ \left. + \frac{b_3 - b_2}{U_C (1+m) (1/Ib_1 - T_C) (b_1 + b_4 - b_2)} + \frac{m(b_3 - b_2)}{U_A (1+m) (m/Ib_4 - T_A) (b_1 + b_4 - b_2)} \right\}^{-1}$$

$$b_1 = (IT_3)^{-1} \quad b_2 = (1+m)T_1^{-1} \quad b_3 = (1+m)T_2^{-1} \quad \text{and} \quad b_4 = m(IT_4)^{-1}$$

The temperature of the working fluid in the generator, evaporator, absorber and condenser which correspond to the maximum ECOP ( $ECOP_{max}$ ) are determined as follows:

$$T_1^* = T_H / [1 + I^{-1/2} \alpha (1-B)]; \quad T_2^* = T_L / [1 + I^{-1/2} \beta (1-B)]; \quad T_3^* = T_C / [1 - \gamma (1-B)]; \quad T_4^* = T_A / B \quad (18)$$

$$\text{Where } B = \frac{z(z-1)T_0^{-1} + (1+m)U_A \xi^{-1} (T_A - T_L + T_C - T_L)^{-1} - (1+m)y(y+I^{1/2})T_L^{-1} - \sqrt{B_1}}{zT_0^{-1} + mT_A^{-1} - (1+m)y^2 T_L^{-1} + (1+m)U_A \xi^{-1} (T_A - T_L + T_C - T_L)^{-1}}$$

$$\alpha = (U_A / U_H)^{1/2} \quad \beta = (U_A / U_L)^{1/2} \quad \gamma = (U_A / U_C)^{1/2}$$

$$B_1 = (1+m)(\beta + \gamma I^{1/2})^2 T_L^{-1} T_C^{-1} - mT_A^{-1} \left[ (1+m)U_A C_1^{-1} + (\gamma - 1)^2 T_C^{-1} - (1+m)(\beta + I^{1/2})^2 T_L^{-1} \right] \\ + (1+m)U_A C_1^{-1} \left[ I(1+m)T_E^{-1} - T_C^{-1} \right]$$

These optimal temperatures obviously lead to the optimal coefficient of performance, the optimal specific cooling load and the optimal specific entropy generation rate at the maximum ECOP conditions for four-heat-source refrigerators.

### 4.1. Four-heat-source heat-pumps

The coefficient of performance

$$COP_{hp} = \frac{\dot{Q}_A + \dot{Q}_C - \dot{Q}_{LC}}{\dot{Q}_H} = \frac{\dot{Q}_C}{\dot{Q}_H} \left( 1 + m - \frac{\dot{Q}_{LC}}{\dot{Q}_C} \right) = \frac{T_1^{-1} - T_2^{-1}}{(IT_3)^{-1} + m(IT_4)^{-1} - (1+m)T_2^{-1}}$$

$$\times \left\{ 1 + m - C_1 \left[ \frac{1}{U_C(T_3 - T_C)} + \frac{m}{U_A(T_4 - T_A)} + \frac{(IT_3)^{-1} + m(IT_4)^{-1} - (1+m)T_2^{-1}}{U_H(T_H - T_1)(T_1^{-1} - T_2^{-1})} + \frac{(IT_3)^{-1} + m(IT_4)^{-1} - (1+m)T_1^{-1}}{U_L(T_L - T_2)(T_2^{-1} - T_1^{-1})} \right] \right\} \quad (19)$$

The specific heating load

$$q = \frac{\dot{Q}_A + \dot{Q}_C - \dot{Q}_{LC}}{A}$$

$$= \frac{1+m}{\frac{1}{U_C(T_3 - T_C)} + \frac{m}{U_A(T_4 - T_A)} + \frac{(IT_3)^{-1} + m(IT_4)^{-1} - (1+m)T_2^{-1}}{U_H(T_H - T_1)(T_1^{-1} - T_2^{-1})} + \frac{(IT_3)^{-1} + m(IT_4)^{-1} - (1+m)T_1^{-1}}{U_L(T_L - T_2)(T_2^{-1} - T_1^{-1})}} - C_1 \quad (20)$$

The specific entropy generation rate

$$s = \frac{\dot{\sigma}}{A} = \frac{1}{A} \left[ \frac{\dot{Q}_C - K_L(T_C - T_E)}{T_C} + \frac{\dot{Q}_A - K_L(T_A - T_E)}{T_A} - \frac{\dot{Q}_G}{T_G} - \frac{\dot{Q}_E - K_L(T_A - T_E + T_C - T_E)}{T_E} \right]$$

$$= C_8 + C_9 \frac{(IT_3)^{-1} + m(IT_4)^{-1} - (1+m)T_2^{-1}}{T_1^{-1} - T_2^{-1}} + \left[ C_6 + C_7 \frac{(IT_3)^{-1} + m(IT_4)^{-1} - (1+m)T_2^{-1}}{T_1^{-1} - T_2^{-1}} \right]$$

$$\times \left[ \frac{1+m}{\frac{1}{U_C(T_3 - T_C)} + \frac{m}{U_A(T_4 - T_A)} + \frac{(IT_3)^{-1} + m(IT_4)^{-1} - (1+m)T_2^{-1}}{U_H(T_H - T_1)(T_1^{-1} - T_2^{-1})} + \frac{(IT_3)^{-1} + m(IT_4)^{-1} - (1+m)T_1^{-1}}{U_L(T_L - T_2)(T_2^{-1} - T_1^{-1})}} - C_1 \right] \quad (21)$$

Where  $C_6 = \frac{1}{1+m} \left( \frac{1}{T_C} + \frac{a}{T_A} \right) - \frac{1}{T_L}$  ;  $C_7 = \frac{1}{1+m} \left( \frac{1}{T_L} - \frac{1}{T_H} \right)$  ;  $C_9 = \frac{\xi(T_A - T_L + T_C - T_L)}{1+m} \left( \frac{1}{T_L} - \frac{1}{T_H} \right)$

$$C_8 = \xi(T_A - T_L + T_C - T_L) \left[ \frac{1}{1+m} \left( \frac{1}{T_C} + \frac{m}{T_A} \right) - \frac{1}{T_L} \right] + \xi \left( \frac{T_A}{T_L} + \frac{T_L}{T_A} + \frac{T_C}{T_L} + \frac{T_L}{T_C} - 4 \right)$$

### The ecological coefficient of performance ECOP

$$ECOP_{hp} = \frac{1}{T_{env}} \left\{ C_6 + \frac{C_7(1+m)(b_1 + b_4 - b_3)}{b_2 - b_3} + \left[ C_8 + \frac{C_9(1+m)(b_1 + b_4 - b_3)}{b_2 - b_3} \right] \right. \\ \left. \times \left[ \frac{1+m}{\frac{Ib_1}{U_C(1-IT_Cb_1)} + \frac{mb_4}{U_A(m-IT_Ab_4)} + \frac{(1+m)(b_1+b_4-b_3)b_2}{U_H(T_Hb_2-1-m)(b_2-b_3)} + \frac{(1+m)(b_1+b_4-b_2)b_3}{U_L(T_Lb_3-1-m)(b_3-b_2)}} - C_1 \right] \right\}^{-1} \quad (22)$$

The temperature of the working fluid in the generator, evaporator, condenser and absorber which correspond to the maximum ECOP are determined as follows:

$$T_1^* = \frac{T_H}{1 + \frac{\alpha}{\sqrt{I}} - \frac{\alpha T_A \sqrt{I}}{m} E_4} ; \quad T_2^* = \frac{T_L}{1 + \frac{\beta}{\sqrt{I}} - \frac{\beta T_A \sqrt{I}}{m} E_4} ; \quad T_3^* = \frac{T_C}{1 - \gamma + \frac{I \gamma T_A}{m} E_4} ; \quad T_4^* = \frac{m}{I E_4} \quad (23)$$

$$\text{where } E_4 = \frac{-(1+m)C_1^{-1}U_A + \beta T_L^{-1}(\beta + \sqrt{I})(1+m) + \gamma T_C^{-1}(1-\gamma) + \sqrt{E}}{-I\gamma^2 m^{-1} T_C^{-1} T_A + (1+m^{-1}) I \beta^2 T_L^{-1} T_A - I - (1+m^{-1}) I U_A C_1^{-1} T_A}$$

$$E = T_L^{-1} T_C^{-1} (1+m) (\gamma \sqrt{I} + \beta)^2 + m T_L^{-1} T_A^{-1} (1+m) (\beta + \sqrt{I})^2 - m T_C^{-1} T_A^{-1} (1-\gamma)^2 \\ - C_1^{-1} U_A (1+m) [m T_A^{-1} + T_C^{-1} - (1+m) I T_L^{-1}]$$

These optimal temperatures obviously lead to the optimal coefficient of performance, the optimal specific heating load and the optimal specific entropy generation rate at the maximum ECOP conditions for four-heat-source heat pumps.

### 5. Results and discussion

Fig. 3 shows the variations of the normalized ECOP objective function (ecop), normalized coefficient of performance (cop) and specific cooling load for three-heat-source absorption refrigerator and the normalized ECOP objective function (ecop), normalized coefficient of performance (cop) and specific heating load for three-heat-source absorption heat pump with respect to specific entropy generation rate. One interesting observation from this figure is that maximum of the ECOP and COP for both the three-heat-source absorption refrigerator and heat pump coincides although their functional forms and their impact on the system design performance are different. The maximum ECOP and COP conditions for both the three-heat-source absorption refrigerator and heat pump give the same amount of  $T_1^*$ ,  $T_2^*$ ,  $T_3^*$ ,  $A_G^*$ ,  $A_L^*$ ,  $A_O^*$ ,  $r^*$  (or  $q^*$ ),  $s^*$  and  $COP^* = COP_{max}$ .

In Fig. 4, the variations of the normalized ECOP objective function (ecop), normalized coefficient of performance (cop) and specific cooling load for four-heat-source absorption refrigerator and the normalized ECOP objective function (ecop), normalized coefficient of performance (cop) and specific heating load for four-heat-source absorption heat pump with respect to specific entropy generation rate are presented. One interesting observation from this figure is that maximum of the ECOP and COP for both the four-heat-source absorption refrigerator and heat pump coincides although their functional forms and their impact on the system design performance are different. The maximum ECOP and COP conditions for both the four-heat-source absorption refrigerator and heat pump give the same amount of  $T_1^*$ ,  $T_2^*$ ,  $T_3^*$ ,  $A_A^*$ ,  $A_L^*$ ,  $A_C^*$ ,  $A_A^*$ ,  $r^*$  (or  $q^*$ ),  $s^*$  and  $COP^* = COP_{max}$ .

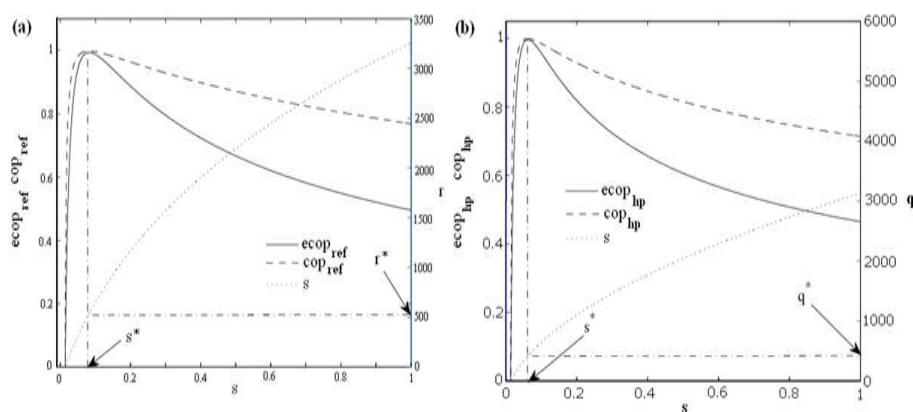


Fig. 3: Variations of the normalized ECOP, normalized COP and the specific cooling load for the three-heat-source refrigerator (a) and Variations of the normalized ECOP, normalized COP and the specific heating load for the three-heat-source heat pump (b) with respect to the specific entropy generation rate

## 6. Conclusion

This work presented investigations on the performance optimization of the irreversible absorption refrigerators and heat pumps. The new thermo-ecological criterion called ecological coefficient of performance, based on the coupling between the first and the second law of thermodynamics and defined as the cooling load or heating load per unit loss rate of availability has been considered and used to carry out the ecological performances of the irreversible absorption refrigerators and heat pumps operating among three or four temperature levels. The optimum performance design parameters, such as the internal working fluid temperatures, the specific cooling load (for refrigerator) and the specific heating load (for heat pump), the coefficient of performance, the specific entropy generation rate and the heat transfer area distributions have been obtained analytically by maximizing the defined thermo-ecological objective function for both the irreversible absorption refrigerators and heat pumps to respect to the internal working fluid temperatures in the main components of the system. Comparisons with the maximum coefficient of performance conditions revealed that the maximum coefficient of performance ( $COP$ ) and the maximum ecological coefficient of performance ( $ECOP$ ) occur for the same operating conditions in spite of their different meaning.

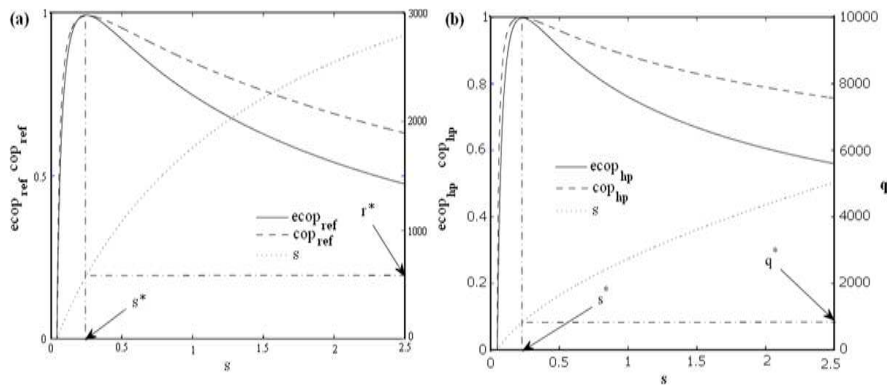


Fig. 4: Variations of the normalized ECOP, normalized COP and the specific cooling load for the four-heat-source refrigerator (a) and Variations of the normalized ECOP, normalized COP and the specific heating load for the four-heat-source heat pump (b) with respect to the specific entropy generation rate

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