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Analysis of a mosquito life cycle model

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Abstract

The gonotrophic cycle of mosquitoes conditions the frequency of mosquito-human contacts. The knowledge of this important phenomenon in the mosquito life cycle is a fundamental element in the epidemiological analysis of a communicable disease such as mosquito-borne diseases. In this work, we analyse a deterministic model of complete life cycle of mosquitoes which takes into account the principal phases of female mosquitoes gonotrophic cycle, and the Sterile Insect technique combined with the use of insecticide as control measures to fight the proliferation of mosquitoes. We compute the corresponding mosquito reproductive number $\mathcal{N}^*$ and prove the global asymptotic stability of trivial equilibrium. We prove that the model admits two non-trivial equilibria whenever $\mathcal{N}^*$ is greater than another threshold, $\mathcal{N}_c$, which the total number of sterile mosquitoes depends on. Numerical simulations, using mosquito parameters of the *Aedes* species, are carried out to illustrate our analytical results and permit to show that the strategy which consists in combining the sterile insect technique with adulticides, when it is well done, effectively combats the proliferation of mosquitoes.

Keywords

Mosquito life cycle; Gonotrophic cycle; Cooperative systems; Sterile Insect Technique (SIT)

I INTRODUCTION

Many diseases among humans are transmitted by mosquitoes and sandflies. According to the World Health Organization (WHO), over a billion people are infected and over a million die from these vector-borne diseases annually [16, 17]. Although some of these diseases have effective vaccines (yellow fever) or effective curative drugs (malaria and lymphatic filariasis), vector-borne diseases remain real public health problems in developing countries. The effective

means of preventing the transmission of most of these vector-borne diseases remain vector control mechanisms. Consequently, a better understanding of the vector life cycle is necessary in order to propose better control mechanisms and thus reduce their proliferation.

The gonotrophic cycle of mosquitoes conditions the frequency of mosquito-human contacts. The knowledge of this important phenomenon in the mosquito life cycle is a fundamental element in the epidemiological analysis of a communicable disease such as mosquito-borne diseases. The frequency of these mosquito-host contacts is indeed dependent on certain factors such as availability of vertebrate hosts, rainfall and temperature.But it depends essentially on the time required for the digestion of the blood meal which is accompanied by the maturation of the ovaries and oviposition and followed by the search for a new blood meal. For Beklemishev [6], the gonotrophic cycle would define the interval between the emergence of the first spawning in the case of nulliparous females and two successive ovipositions in the case of parous females (see also [7]). In general, the gonotrophic cycle consists of following distinct phases [5, 11]:

1. blood meal seeking: search of a vertebrate host by a young female mosquito after its mating with a male mosquito, or an adult female mosquito which has just laid eggs in a breeding site,
2. blood meal digesting: digestion of ingested blood and ovarian maturation,
3. gravid: laying eggs by pregnant females (gravid) in a favourable breeding site,
4. search of another blood meal, . . .

Figure 1 presents the principal stages of mosquito life cycle. The time between mating and a

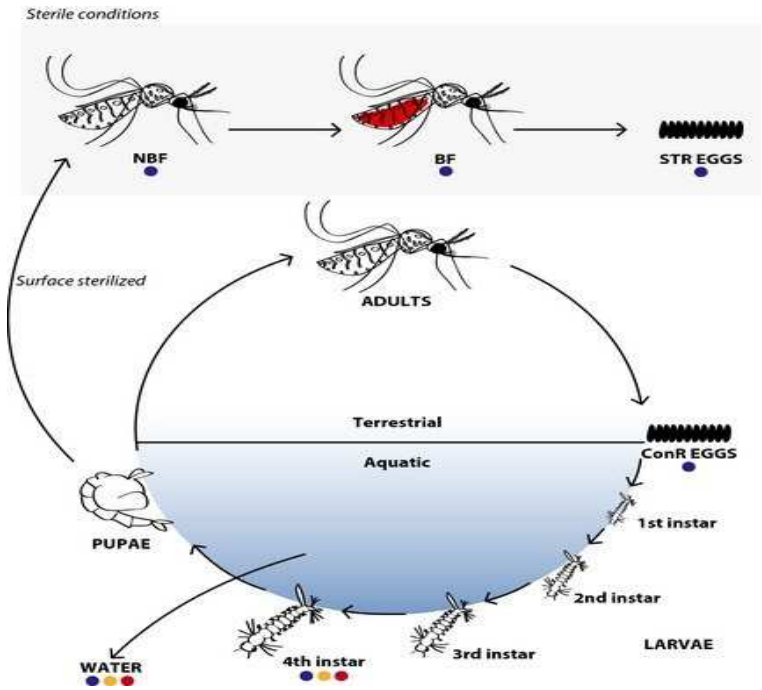

Figure 1: Life cycle of female mosquitoes [8].

successful blood meal seeking is a key factor in the female mosquito life who increases their mortality rate in the finding of blood to ensure development of eggs. For many mosquitoes species such as *Anopheles sp.*, the search of a blood meal is between 06 pm to 06 am [5, 11]. Thus, it is in this time interval that some vector control mechanisms such as treated insecticide bed-nets usage and insecticides permit to decrease the number of mosquito-human contacts or increase the mosquito death rate. The technique called sterile insect technique (SIT), was

developed to stop the mosquitoes proliferation [9, 11]. Indeed, this mosquito control mechanism consists to release around breeding sites, male mosquitoes which was irradiated, in order for them to compete with wild male mosquitoes and mate with female mosquitoes. So, the eggs laying by female mosquitoes who had mated with sterile males will not be able to hatch. Thus, the next generation of mosquito population will decrease according to the number of sterile males mosquitoes whom mated with wild female mosquitoes. This technique has been used to control the proliferation of *Aedes albopictus* population during the 2005 chikungunya epidemic in Reunion Island [9, 10].

There is a long history in the mathematical modelling of life cycle of mosquitoes. After the chikungunya epidemic outbreak in 2005, many mathematical models, which take into account some principal stages of mosquito life cycle, were proposed (see for example [1, 2, 3, 5, 9, 10, 11, 13, 18, 19]). In [3, 9, 10], the authors propose some various models with aquatic development phase of *Aedes sp.* mosquitoes coupled with the transmission dynamics of chikungunya virus in human populations. To prove the impact of sterile insect techniques on the decrease of *Aedes* population during the 2005 chikungunya epidemic in Reunion island, Dumont et *al.* [3, 9] proposed and studied two compartmental models. They did not modelled the complete mosquito life cycle. In the same idea, Moulay et *al.* [13] (see also Yang et *al.* [18] and Yusoff et *al.* [19]) proposed also a compartmental model in which they modelled eggs and larvae to represent mosquito aquatic phase of *Aedes albopictus*. They did not take into account the pupae stage and the gonotrophic cycle of mosquitoes. Abboubakar et *al.* [1, 2] extended the models of Dumont et *al.* [9] and Moulay et *al.* [13] by adding the pupa stage in their models. Arifin et *al.* [5] and Gentile et *al.* [11] proposed two agent-based models to study the dynamic behavior of *Anopheles gambiae*. Here, we present the corresponding ODE model of Arifin et *al.* [5] and Gentile et *al.* [11] by the representation of each mosquito life cycle phase in a different compartment, and in which we include the sterile insect technique and the use of insecticide to decrease the population of mosquitoes. We prove the global stability of the trivial equilibrium and show that whenever the mosquito reproductive number $\mathcal{N}^*$ is greater than one and greater than an other threshold, $\mathcal{N}_c$, which depends on the total number of sterile mosquitoes, the model admits two positive equilibria. Numerical simulations are performed to illustrate our analytical results.

The paper is organised as follow. The description of the complete life cycle model of mosquitoes (including gonotrophic cycle, SIT and the use of insecticide) and its mathematical analysis are devoted in Section II. Numerical simulations are performed in Section III.

II THE MODEL AND ITS ANALYSIS

Here, we describe the general mosquito life cycle model.

2.1 Aquatic phase

Aquatic phase is divided in three compartments: E for eggs, L for larvae and P for pupae. Eggs are laid with a rate μ_b. After 2 to 4 days depending of ambient temperature, eggs become larvae at a rate s. The larvae will in turn become pupae at a rate l. μ_E, μ_L and μ_P denotes the mortality rate of eggs, larvae and pupae, respectively. We assume in this work that, based on the larval population size, the density-dependence affect the survival rates of both larvae and pupae with linear density functions αL and βL, respectively [4].

2.2 Juvenile stage and mosquito ready to mate

They are two compartments for juvenile mosquitoes. We denote by X the total number of male mosquitoes and by Y the total number of female mosquitoes. Indeed, after the completed aquatic phase, the pupae will become either male mosquitoes X at a rate $(1-\vartheta)\eta$ or female mosquitoes Y at a rate $\vartheta\eta$, where ϑ is the sex ratio and η is the pupae development rate. Then comes the phase where a male mosquito will mate with a female mosquito F at a constant rate β_2. We denote by μ_X, μ_Y and μ_F the mortality rates of male mosquitoes, juvenile female mosquitoes and adult female mosquitoes, respectively.

2.3 The gonotrophic cycle

We subdivide the gonotrophic cycle in three compartments: S for total number of female mosquitoes in blood meal seeking stage, D for total number of female mosquitoes in blood meal digesting stage, and G for total number of female mosquitoes in gravid stage. After mating with a male, female mosquito will go to search a vertebrate host for a blood meal rich in protein which will ensure the proper development of their eggs, at a rate ω. Female mosquitoes in blood meal seeking stage die at constant rate μ_S. After a successful blood meal, seeking mosquitoes find a place to digest at constant rate γ_1. Female mosquitoes in blood meal digesting stage die at constant rate μ_D, and become gravid G at the constant rate γ_2. Gravid mosquitoes die at a constant rate μ_G. In this stage, gravid mosquitoes find an appropriate breeding site to lay their eggs, and search another vertebrate host to find a new blood meal at a rate ϵ, and then enter in another gonotrophic cycle.

The above assumptions lead to the following nonlinear system of ordinary differential equations

$$\dot{E}(t) = \mu_b G(t) - (s+\mu_E)E(t), \tag{1a}$$
$$\dot{L}(t) = sE(t) - (l+\mu_L+\alpha L(t))L(t), \tag{1b}$$
$$\dot{P}(t) = lL(t) - (\eta+\mu_P+\beta_1 L(t))P(t), \tag{1c}$$
$$\dot{X}(t) = (1-\vartheta)\eta P(t) - \mu_X X(t) \tag{1d}$$
$$\dot{Y}(t) = \vartheta\eta P(t) - (\mu_Y+\beta_2)Y(t), \tag{1e}$$
$$\dot{F}(t) = \beta_2 Y(t) - (\mu_F+\omega)F(t), \tag{1f}$$
$$\dot{S}(t) = \omega F(t) + \epsilon G(t) - (\mu_S+\gamma_1)S(t), \tag{1g}$$
$$\dot{D}(t) = \gamma_1 S(t) - (\mu_D+\gamma_2)D(t), \tag{1h}$$
$$\dot{G}(t) = \gamma_2 D(t) - (\mu_G+\epsilon)G(t). \tag{1i}$$

Without loss of generalities, we claim the following results.

Lemma 1:
Every solution $\psi(t) = ((E(t), L(t), P(t), X(t), Y(t), F(t), S(t), D(t), G(t)))$ of (1) with its initial conditions in $\mathbb{R}^9_+$ is defined and lies in $int(\mathbb{R}^9_+)$ for all $t > 0$.

Proof. Let f be the right-hand side of (1) and $k_1 = s+\mu_E$, $k_2 = l+\mu_L$, $k_3 = \mu_P+\eta$, $k_4 = \beta_2+\mu_Y$, $k_5 = \mu_F+\omega$, $k_6 = \mu_S+\gamma_1$, $k_7 = \mu_D+\gamma_2$ and $k_8 = \mu_G+\varepsilon$. The Jacobian

matrix of f is

$$Df(E,L,P,X,Y,F,S,D,G) = \begin{pmatrix} -k_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_b \\ s & -(k_2+2\alpha L) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & l-\beta_1 P & -(k_3+\beta_1 L) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & (1-\vartheta)\eta & -\mu_X & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \vartheta\eta & 0 & -k_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta_2 & -k_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \omega & -k_6 & 0 & \varepsilon \\ 0 & 0 & 0 & 0 & 0 & 0 & \gamma_1 & -k_7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_2 & -k_8 \end{pmatrix}. \tag{2}$$

To prove Lemma 1, we proceed by step. Since (1) is a smooth system in $\mathbb{R}^9$, $\psi(t)$ exists on a maximal interval $[0, T^+)$ for some $0 < T^+ \leq \infty$.

Step 1 We show that $\psi(t) > 0$ for $t \in [0, T^+)$. Using the variation of parameters formula, we have, for $t \in [0, T^+)$

$$\begin{aligned} E(t) &= E(0)e^{-k_1 t} + \mu_b \int_0^t G(\tau)e^{-k_1(t-\tau)}d\tau, \\ L(t) &= L(0)e^{-\int_0^t (k_2+\alpha L(\tau))d\tau} + s\int_0^t E(\tau)e^{-\int_\tau^t (k_2+\alpha L(\eta))d\eta}d\tau, \\ P(t) &= P(0)e^{-\int_0^t (k_3+\beta_1 L(\tau))d\tau} + l\int_0^t L(\tau)e^{-\int_\tau^t (k_3+\beta_1 L(\eta))d\eta}d\tau, \\ X(t) &= X(0)e^{-\mu_X t} + (1-\vartheta)\eta\int_0^t P(\tau)e^{-\mu_X(t-\tau)}d\tau, \\ Y(t) &= Y(0)e^{-k_4 t} + \vartheta\eta\int_0^t P(\tau)e^{-k_4(t-\tau)}d\tau, \\ F(t) &= F(0)e^{-k_5 t} + \beta_2\int_0^t Y(\tau)e^{-k_5(t-\tau)}d\tau, \\ S(t) &= S(0)e^{-k_6 t} + \omega\int_0^t F(\tau)e^{-k_6(t-\tau)}d\tau + \varepsilon\int_0^t G(\tau)e^{-k_6(t-\tau)}d\tau, \\ D(t) &= D(0)e^{-k_7 t} + \gamma_1\int_0^t S(\tau)e^{-k_7(t-\tau)}d\tau, \\ G(t) &= G(0)e^{-k_8 t} + \gamma_2\int_0^t D(\tau)e^{-k_8(t-\tau)}d\tau, \end{aligned} \tag{3}$$

Since $\psi(t) \in \mathbb{R}^9_+ \setminus \{0\}$, we have $E(0) > 0$, $L(0) > 0$, $P(0) > 0$, $X(0) > 0$, $Y(0) > 0$,$F(0) > 0$,$S(0) > 0$, $D(0) > 0$ or $G(0) > 0$. If $E(0) > 0$, it follows from the continuity of E that $E(t) > 0$, for $t \in (0, \epsilon)$, for small $\epsilon > 0$. Then it follows from (3) that $L(t) > 0$ on $(0, \epsilon]$, which, in turn, implies that $P(t) > 0$, $X(t) > 0$, $Y(t) > 0$,$F(t) > 0$, $S(t) > 0$, $D(t) > 0$ on $(0, \epsilon]$, and then $G(t) > 0$ on $(0, \epsilon]$. Note that for any $t \in [\epsilon, T^+)$, as long as E, L, P, X, Y, F, S, D, G are positive on $(0, t)$, all the integrals on the right-hand side of (3) are positive. It then follows from (3) that $E(t) > 0$, $L(t) > 0$, $P(t) > 0$, $X(t) > 0$, $Y(t) > 0$, $F(t) > 0$, $S(t) > 0$, $D(t) > 0$ and $G(t) > 0$, for any $t \in [\epsilon, T^+)$, as long as E, L, P, X, Y, F, S, D,G, are positive on $(0, t)$. This yields that E, L, P, X_1, Y, F, S, D and G are positive on $(0, T^+)$. The same assertion holds in the cases that $L(0) > 0$, $P(0) > 0$,$X(0) > 0$, $Y(0) > 0$, $F(0) > 0$, $S(0) > 0$, $D(0) > 0$ or $G(0) > 0$.

Step 2 We next show that $T^+ = +\infty$. To this aim, we compare $\psi(t)$ with a solution of the variational system of (1) at its trivial equilibrium

$$\dot{z} = Df(0)z \tag{4}$$

which is a cooperative system in $\mathbb{R}^9$. Let $z(t)$ be the solution of (4), with $z(0) = \psi(0)$. Since $\psi(t) \in \mathbb{R}^9_+ \setminus \{0\}$, for $t \in [\epsilon, T^+)$, it follows, for $t \in [\epsilon, T^+)$, that

$$\dot{\psi}(t) = f(\psi(t)) = Df(0)\psi(t) - \big(0, \alpha L^2(t), \beta_1 L(t)P(t), 0, 0, 0, 0, 0, 0\big)^T \leq Df(0)\psi(t).$$

Then, from Theorem 4, it follows that $\psi(t) \leq z(t)$, for $t \in [0, T^+)$. Since $z(t)$ is defined for all $t \geq 0$, it follows that $\psi(t)$ is defined for all $t \in (0, +\infty)$, i.e. $T^+ = +\infty$.

The results from Steps 1-2 permit us to conclude the proof of lemma 1. □

Define the net reproductive number [13], given by

$$\mathcal{N}^* = \frac{s}{(s+\mu_E)}\frac{l}{(l+\mu_L)}\frac{\vartheta\eta}{(\eta+\mu_P)}\frac{\mu_b\beta_2}{(\beta_2+\mu_Y)}\frac{\omega}{(\omega+\mu_F)}\frac{\gamma_1\gamma_2}{(k_6k_7k_8-\varepsilon\gamma_1\gamma_2)}, \tag{5}$$

where $k_6k_7k_8 - \epsilon\gamma_1\gamma_2 = \gamma_1\mu_D\epsilon + (\mu_D+\gamma_2)(\mu_G(\mu_S+\gamma_1)+\mu_S\epsilon) > 0$.

$\mathcal{N}^*$ measures the average expected number of new adult female offsprings produced by a single female vector during its life time. It can be interpreted as the product of the fraction of eggs that survived and hatched into larvae $s/(s+\mu_E)$, the fraction of larvae that survived and progressed into pupae $l/(l+\mu_L)$, the fraction of pupae that survived to become juvenile female mosquitoes $\vartheta\eta/(\eta+\mu_P)$, the fraction of juvenile female mosquitoes that survived to become adult female mosquitoes $\mu_b\beta_2/(\beta_2+\mu_Y)$, the fraction of the adult females which survived to give mosquitoes in search of blood meal $\omega/(\omega+\mu_F)$, the fraction of the mosquitoes in search of blood meal which survived to give mosquitoes in gravid (pregnant) state $\gamma_1\gamma_2/(k_6k_7k_8-\epsilon\gamma_1\gamma_2)$.

To decrease the proliferation of mosquito population, we use the biological mosquito control called sterile insect technique (SIT) combined with the use of insecticide and treated insecticide bed-nets which permits to increase the mortality rate of mosquitoes in blood meal seeking stage. To this aim, we firstly add one compartment X_2 of sterile male mosquitoes, which would compete with wild male mosquitoes to mate with female mosquitoes. Sterile males are released into the wild in an amount 5 to 10 times greater than that of wild males [9, 10]. It only takes one mating of a female mosquito with a sterile male to reduce their fertility since it stores the sperm of the first male which fertilized their to conceive all their eggs throughout their life (about a month). Secondly, we modify the mortality rate of mosquitoes in blood meal seeking stage by adding a constant c_m which represents the insecticide killing rate ($\mu_S := \mu_S + c_m$). Thus, the complete life cycle mosquito model with the introduction of sterile male compartment is as follow.

$$\dot{E}(t) = \mu_b G(t) - (s+\mu_E)E(t), \tag{6a}$$
$$\dot{L}(t) = sE(t) - (l+\mu_L+\alpha L(t))L(t), \tag{6b}$$
$$\dot{P}(t) = lL(t) - (\eta+\mu_P+\beta_1 L(t))P(t), \tag{6c}$$
$$\dot{X}_1(t) = (1-\vartheta)\eta P(t) - \mu_{X_1}X_1(t), \tag{6d}$$
$$\dot{X}_2(t) = fr\Lambda_1 - \mu_{X_2}X_2(t), \tag{6e}$$
$$\dot{Y}(t) = \vartheta\eta P(t) - (\mu_Y+\beta_2)Y(t), \tag{6f}$$
$$\dot{F}(t) = \beta_2\left(\frac{eX_1(t)}{eX_1(t)+\pi X_2(t)}\right)Y(t) - (\mu_F+\omega)F(t), \tag{6g}$$
$$\dot{S}(t) = \omega F(t) + \epsilon G(t) - (\mu_S+\gamma_1+c_m)S(t), \tag{6h}$$
$$\dot{D}(t) = \gamma_1 S(t) - (\mu_D+\gamma_2)D(t), \tag{6i}$$
$$\dot{G}(t) = \gamma_2 D(t) - (\mu_G+\epsilon)G(t). \tag{6j}$$

In Eq. (6), $fr\Lambda_1$ represent the efficient release of sterile males with f which denotes the competitivity of the sterile male, r is a parameter that represents the quality of the release, and Λ_1 denotes the rate of release of sterile males [10]. It is important to note that we follow Dumont and Tchuenche [10] and suppose that the introduction of sterile males only impacts the incoming of immature females in the wild females compartment F.

2.4 The trivial equilibrium and its stability analysis

The model system (6) admits a trivial equilibrium $\mathcal{Q}_1 = (0, 0, 0, 0, X_2^*, 0, 0, 0, 0, 0)$ whenever $\mathcal{N}^* \leq 1$, where $\mathcal{N}^*$ is the net reproduction number given by (5) with $c_m = 0$, and $X_2^* = \frac{fr\Lambda_1}{\mu_{X_2}}$.

The stability analysis of the trivial equilibrium of model system (6) is given by the following result.

Theorem 1:
The trivial equilibrium point $\mathcal{Q}_1 = (0, 0, 0, 0, X_2^*, 0, 0, 0, 0, 0)$ of (6) is globally asymptotically stable in $\mathbb{R}_+^{10}$ if $\mathcal{N}^* \leq 1$, and unstable if $\mathcal{N}^* > 1$.

Proof. The Jacobian matrix of system (6) at the trivial equilibrium $\mathcal{Q}_1$, with the components being reordered to $\mathcal{Q} = (E, L, P, X_1, X_2, Y, F, S, D, G)$, has the following form

$$\mathcal{J}(\mathcal{Q}_1) = \begin{pmatrix} \mathcal{A}_{11} & \mathcal{A}_{12} \\ 0 & \mathcal{A}_{22} \end{pmatrix},$$

where $\mathcal{A}_{11} = \begin{pmatrix} -k_1 & 0 & 0 & 0 & 0 & 0 \\ s & -k_2 & 0 & 0 & 0 & 0 \\ 0 & l & -k_3 & 0 & 0 & 0 \\ 0 & 0 & (1-\vartheta)\eta & -\mu_{X_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\mu_{X_2} & 0 \\ 0 & 0 & \vartheta\eta & 0 & 0 & -k_4 \end{pmatrix}$,

$\mathcal{A}_{12} = \begin{pmatrix} 0 & 0 & 0 & \mu_b \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ and $\mathcal{A}_{22} = \begin{pmatrix} -k_5 & 0 & 0 & 0 \\ \omega & -k_6 & 0 & \epsilon \\ 0 & \gamma_1 & -k_7 & 0 \\ 0 & 0 & \gamma_2 & -k_8 \end{pmatrix}$.

The eigenvalues of $\mathcal{J}(\mathcal{Q}_1)$ consist of those of $\mathcal{A}_{11}$ and $\mathcal{A}_{22}$. The eigenvalues of $\mathcal{A}_{11}$ are given by $\lambda_1 = -k_1$, $\lambda_2 = -k_2$, $\lambda_3 = -k_3$, $\lambda_4 = -\mu_{X_1}$, $\lambda_5 = -\mu_{X_2}$ and $\lambda_6 = -k_4$, which are all negative. It easy to see , using Routh- criterion that $\mathcal{A}_{22}$ has eigenvalues with negative real part whenever $\mathcal{N} \leq 1$, where $\mathcal{N} = \frac{s}{(s+\mu_E)} \frac{l}{(l+\mu_L)} \frac{\vartheta\eta}{(\eta+\mu_P)} \frac{\mu_b\beta_2}{(\beta_2+\mu_Y)} \frac{1}{\mu_F}$. For the global stability, let us consider the following Lyapunov function

$$\mathcal{L}(x, X_2) = \sum_{i=1}^{9} a_i x_i + \frac{1}{2}(X_2 - X_2^*)^2, \tag{7}$$

where $x = (E, L, P, X_1, Y, F, S, D, G)$ and $a_1 = k_4k_5k_9\mathcal{N}^*$, $a_2 = k_1k_4k_5k_9\mathcal{N}^*/s$, $a_3 = k_1k_2k_4k_5k_9\mathcal{N}^*/sl$, $a_4 = \mathcal{N}^*\beta_2\vartheta\mu_b\omega\gamma_1\gamma_2$, $a_5 = \mathcal{N}^*\beta_2\vartheta\mu_b\omega\gamma_1\gamma_2$, $a_6 = \mathcal{N}^*k_4\vartheta\mu_b\omega\gamma_1\gamma_2$, $a_7 = \mathcal{N}^*k_4k_5\vartheta\mu_b\gamma_1\gamma_2$, $a_8 = \mathcal{N}^*k_4k_5k_6\vartheta\mu_b\gamma_2$ and $a_9 = \mathcal{N}^*k_4k_5k_6k_7\vartheta\mu_b$.

The Lyapunov derivative of (7) is given by

$$\begin{aligned}\dot{\mathcal{L}}(x, X_2) &= \sum_{i=1}^{9} a_i \dot{x}_i + (X_2 - X_2^*)\,\dot{X}_2 \\ &= -(k_1k_4k_5k_9\mathcal{N}^*/s)\alpha L^2 - (k_1k_2k_4k_5k_9\mathcal{N}^*/sl)\beta_1 LP - \mu_{X_2}(X_2 - X_2^*)^2 \\ &+ \frac{k_1k_2k_3k_4k_5k_9\mathcal{N}^*}{sl}(\mathcal{N}^* - 1)P - \mathcal{N}^*\beta_2\vartheta\mu_b\omega\gamma_1\gamma_2\mu_{X_1}X_1 \\ &- \mathcal{N}^*k_4\beta_2\vartheta\mu_b\omega\gamma_1\gamma_2 Y + \mathcal{N}^*k_4\vartheta\mu_b\omega\gamma_1\gamma_2\beta_2\left(\frac{eX_1}{eX_1 + \pi X_2}\right)Y \\ &\leq -(k_1k_4k_5k_9\mathcal{N}^*/s)\alpha L^2 - (k_1k_2k_4k_5k_9\mathcal{N}^*/sl)\beta_1 LP - \mu_{X_2}(X_2 - X_2^*)^2 \\ &- \mathcal{N}^*\beta_2\vartheta\mu_b\omega\gamma_1\gamma_2\mu_{X_1}X_1 + \frac{k_1k_2k_3k_4k_5k_9\mathcal{N}^*}{sl}(\mathcal{N}^* - 1)P\end{aligned} \tag{8}$$

It follows that $\dot{\mathcal{L}}(x, X_2) \leq 0$ for $\mathcal{N}^* \leq 1$. The largest compact invariant set in $\{(E, L, P, X_1, X_2, Y, F, S, D, G) \in \mathbb{R}^+_{10} : \dot{\mathcal{L}}(x, X_2) = 0\}$ is the singleton $\{\mathcal{Q}_1\}$. It follows from the LaSalle Invariance Principle [12, Chapter 2, Theorem 6.4] that every solution to the equations in (6) with initial conditions in $\mathbb{R}^+_{10}$ converge to the trivial equilibrium $\mathcal{Q}_1$ as $t \longrightarrow +\infty$. Thus $(x(t), X_2(t)) \longrightarrow (0_{\mathbb{R}^9}, X_2^*)$ when $t \longrightarrow +\infty$ for $\mathcal{N}^* \leq 1$. So, From the LaSalle principle we deduce the attractiveness of $\mathcal{Q}_1$. Since $\mathcal{Q}_1$ is locally asymptotically stable when $\mathcal{N}^* \leq 1$, we deduce that it is not only attractive, but it is also globally asymptotically stable. This ends the proof. □

2.5 The non-trivial equilibria

In the following, we consider that the net reproductive number $\mathcal{N}^*$ is greater than unity. We have the following result.

Theorem 2:
Let us define the following thresholds:

$$\xi = \frac{\Lambda_1 fr}{\mu_{X_2}}\frac{\mu_{X_1}\pi\,(\alpha k_3 + 2\beta_1 k_2)}{el(1-\vartheta)\eta k_2} \quad \text{and} \quad \mathcal{N}_c = 1 + \xi. \tag{9}$$

Assume that $\mathcal{N}^* > 1$.

1. If $\mathcal{N}^* \leq \mathcal{N}_c$, model system (6) has no positive equilibrium point.
2. If $\mathcal{N}^* > \mathcal{N}_c$, model system (6) admits two positive equilibrium points $\mathcal{Q}_2^{+/-} = \left(E^*_{+/-}, L^*_{+/-}, P^*_{+/-}, X^*_{1+/-}, X^*_2, Y^*_{+/-}, F^*_{+/-}, S^*_{+/-}, D^*_{+/-}, G^*_{+/-}\right)$ where

$$\begin{aligned}&X_2^* = \frac{fr\wedge_1}{\mu_{X_2}}, \quad P^* = \frac{lL^*_{+/-}}{k_3 + \beta_1 L^*_{+/-}}, \quad Y^*_{+/-} = \frac{\vartheta\eta}{k_4}P^*_{+/-}, \quad X^*_{1_{+/-}} = \frac{(1-\vartheta)\eta}{\mu_{X_1}}P^*_{+/-}, \\ &F^*_{+/-} = \frac{\beta_2 eX^*_{1+/-}}{k_5(eX^*_{1+/-} + \pi X^*_2)}Y^*_{+/-}, \quad S^*_{+/-} = \frac{\omega k_7 k_8}{k_9}F^*_{+/-}, \quad D^*_{+/-} = \frac{\gamma_1}{k_7}S^*_{+/-}, \\ &G^*_{+/-} = \frac{\gamma_2}{k_8}D^*_{+/-}, \quad E^*_{+/-} = \frac{\mu_b}{k_1}G^*_{+/-},\end{aligned} \tag{10}$$

and $L^*_{+/-}$ are positive solutions of the following equation

$$\mathcal{P}(L^*) = L^* \left[\rho_3 L^{*3} + \rho_2 L^{*2} + \rho_1 L^* + \rho_0\right] = 0, \tag{11}$$

with $\rho_3 = -\alpha\beta_1 e\eta l\vartheta\mu_{X_2} - \Lambda_1\alpha\beta_1^2 f\mu_{X_1}\pi r < 0$,
$\rho_2 = -\left[e\eta\vartheta l\mu_{X_2}(\alpha k_3 + \beta_1 k_2) + \Lambda_1\beta_1 f\mu_{X_1}\pi r(2\alpha k_3 + \beta_1 k_2)\right] < 0$,
$\rho_1 = el(1-\vartheta)\eta\mu_{X_2}k_2k_3\left[\mathcal{N}^* - \mathcal{N}_c\right]$ and $\rho_0 = -\Lambda_1 f k_2 k_3^2 \mu_{X_1}\pi r < 0$.

Proof. By setting the hand right sides of the equations of (6) to zero and expressing the variables in terms of L, we obtain that the non-trivial equilibria come from the resolution of (11) in terms of L. It follows, using Descartes' Rule of Signs, that the polynomial (11) has two positive non-trivial roots (L_1^+ and L_2^-) whenever $\mathcal{N}^* > \mathcal{N}_c$, and no positive root otherwise. Thus, we conclude that (6) admits two non-trivial positive equilibria ($\mathcal{Q}_2^+$ and $\mathcal{Q}_2^-$) whenever $\mathcal{N}^* > \mathcal{N}_c$. This ends the proof. □

It is important to note that, in the presence of sterile males, the condition $\mathcal{N}^* > 1$ is not sufficient to the mosquitoes proliferation (see for example [1, 2, 3, 10, 13]). Indeed, it follows from the item (1) of Theorem 2, that if $1 < \mathcal{N}^* \leq \mathcal{N}_c$, there is no non-trivial equilibrium, which implies that there are no mosquitoes. This is illustrated in figure 3.

Note that in absence of sterile males, i.e $\Lambda_1 = 0$ or $f = r = 0$, equation (11) becomes $(L^*)^2\left[\rho_2 L^{*2} + \rho_1 L^* + \rho_0\right] = 0$, with $\rho_2 = -\alpha\beta_1 e\eta l\mu_{X_2}\vartheta < 0$,
$\rho_1 = -\left(\alpha e\eta k_3 l\mu_{X_2}\vartheta + \beta_1 e\eta k_2 l\mu_{X_2}\vartheta\right) < 0$, and $\rho_0 = el(1-\vartheta)\eta\mu_{X_2}k_2k_3\left[\mathcal{N}^* - 1\right]$. So, the system admits only one non-trivial equilibrium whenever $\mathcal{N}^* > 1$.
Now, consider that $\mathcal{N}^* > 1$. Then, if the following condition holds

$$\rho_1 > 0 \Longleftrightarrow \mathcal{N}_c < \mathcal{N}^* \Longleftrightarrow \Lambda_1 < \Lambda_1^{crit} = \frac{e}{\pi}\frac{\mu_{X_2}}{fr}\frac{l(1-\vartheta)\eta}{\mu_{X_1}}\frac{k_2}{(\alpha k_3 + 2\beta_1 k_2)}\left(\mathcal{N}^* - 1\right), \tag{12}$$

model system (6) admits two non-trivial equilibria $\mathcal{Q}_2^+$ and $\mathcal{Q}_2^-$. Then, we can reformulate theorem 2 as follows.

Theorem 3:
Assume that $\mathcal{N}^* > 1$. Then there exist $\Lambda_1^{crit} > 0$ such that model (6) admits two non-trivial equilibria if $0 < \Lambda_1 < \Lambda_1^{crit}$ and no non-trivial equilibrium otherwise.

III NUMERICAL SIMULATIONS

In this subsection, we perform some numerical simulations to illustrate results of theorem 3 and 2. To this aim, we use the following parameter values $\mu_b = 250$, $s = 0.7$, $\mu_E = 0.2$, $\mu_L = 0.4$, $\mu_P = 0.4$, $l = 0.5$, $\omega = 0.8$, $\vartheta = 0.5$, $f = 0.7$, $r = 0.5$, $\alpha = 0.07$, $\eta = 0.5$, $\beta_1 = 0.08$, $\mu_{X_1} = 1/14$, $\mu_{X_2} = 1/8$, $\beta_2 = 0.7$, $\mu_Y = 1/15$, $e = 0.5$, $\mu_F = 1/15$, $\epsilon = 0.8$, $\mu_S = 1/10$, $\mu_D = 1/14$, $\gamma_1 = 1/2$, $\gamma_2 = 1/2$, $\mu_G = 1/12$ and $c_m = 0$ [1, 10]. We obtain $\mathcal{N}^* = 61.4689 > \mathcal{N}_c = 13.144$, $\Lambda_1 = 33 < \Lambda_1^{crit} = 164.31766$, $\rho_3 = -2.285 \times 10^{-4} < 0$, $\rho_2 = -7.5887 \times 10^{-4} < 0$, $\rho_1 = 0.305806 > 0$ and $\rho_0 = -0.3007125 < 0$. So conditions of theorems 2 and 3 hold. The positive solutions of equation (11) are $L_+^* = 23$ and $L_-^* = 1$. Figure 2(a) shows clearly existence of two positive solution of equation (11) whenever condition (12) holds. From figure 2(b) we see that one of the two positive equilibria, $\mathcal{Q}_2^+$, is locally stable. Figure 3 illustrate the situation which happens when the condition of item (1) of Theorem 2

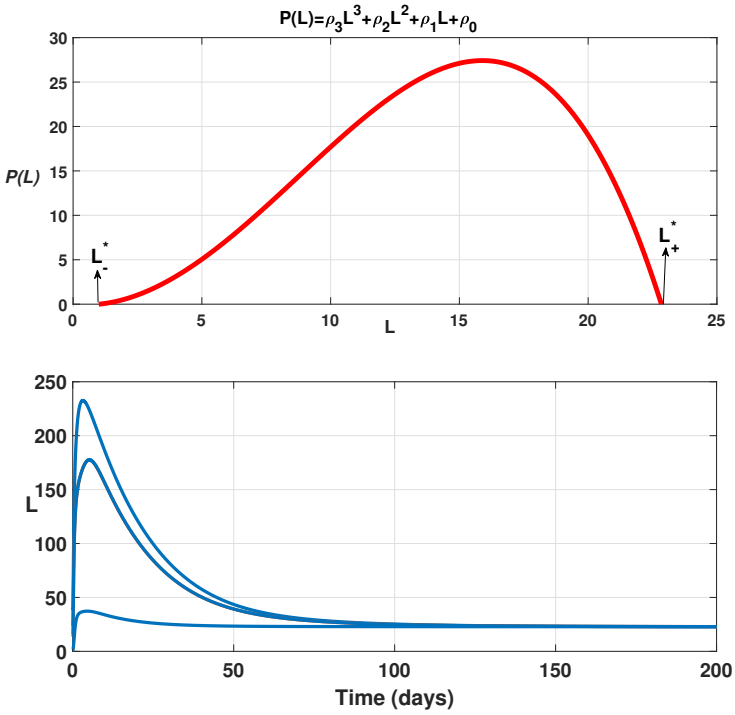

Figure 2: Illustration of theorem 3 for $1 < \mathcal{N}^* = 61.4689 > \mathcal{N}_c = 13.144$ and $\Lambda_1 = 33 < \Lambda_1^{crit} = 164.31766$. So, the positive equilibrium $\mathcal{Q}_2^+$ is locally stable.

holds, i.e. when $1 < \mathcal{N}^* \leq \mathcal{N}_c$. Indeed, it clear that if there is this condition holds, then there are no non-trivial equilibria, which implies that there are no mosquitoes.

The effects of varying parameter e to the decrease of the total number larvae are depicted on figure 4. It is clear that if female mosquitoes have a preference to mate with sterile males, the population of larvae will decrease significantly. Now, we numerically study the impact of sterile insect technique combined with the use of conventional insecticide on larvae and mosquitoes in blood meal seeking stage. Because Deltamethrin, for example, is effective only during a couple of hours and taking into account the preservation of the environment, it is not realistic to spray this chemical product continuously. So, we use "pulse control" technique which means that "the control is not continuous in time order is effective only one day every T day" [9]. Also, the continue release of sterile males mosquitoes is not realistic. We thus consider that sterile

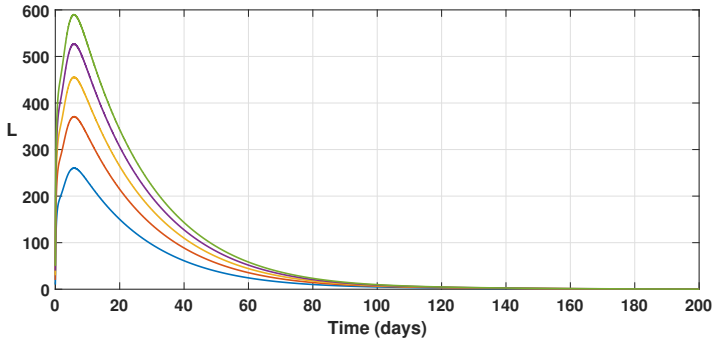

Figure 3: Illustration of item (1) of theorem 3 for $1 < \mathcal{N}^* = 12.2938 \leq \mathcal{N}_c = 15.6194$. Parameter values are the same that those use in figure 2. So, model system (1) has no non-trivial equilibrium, which means that there are no mosquitoes where the condition $1 < \mathcal{N}^* \leq \mathcal{N}_c$ holds.

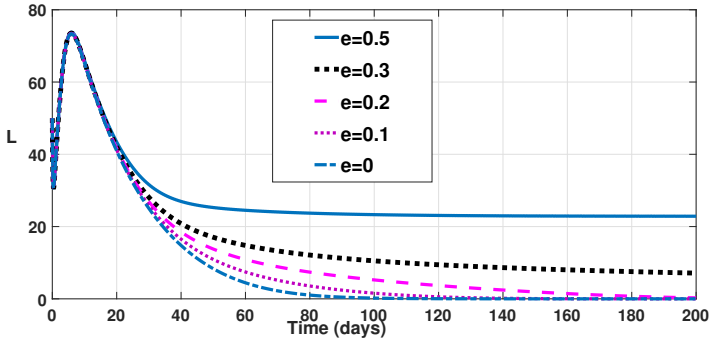

Figure 4: Effect of preference of female mosquitoes to mate with sterile males on larvae.

males will be release after every one week (seven days) [9, 10]. Using the above assumptions in numerical simulations, we obtain the following figures on which we clearly see the impact of the use of these two controls technique on larvae and mosquitoes in blood meal seeking stage. From figure 5, it follows that increases in the number of sterile males permit to decrease in both larvae and female mosquitoes in the blood meal seeking. The same observation can be done if we use insecticide (adulticide). Indeed, in figure 6, it is clear that the periodic use of adulticide permits to decrease both larvae and female mosquitoes in the blood meal seeking. From figure 7, it is clear that the combination of these two controls permits to reduce significantly the total number of larvae and mosquitoes in the blood meal seeking stage.

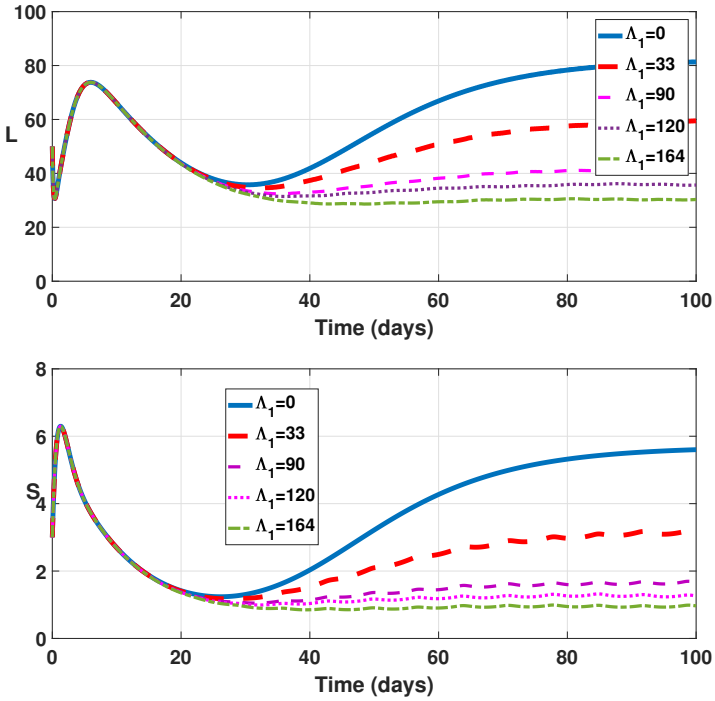

Figure 5: Effect of release sterile male mosquitoes after every one week on larvae L and blood meal seeking mosquitoes S.

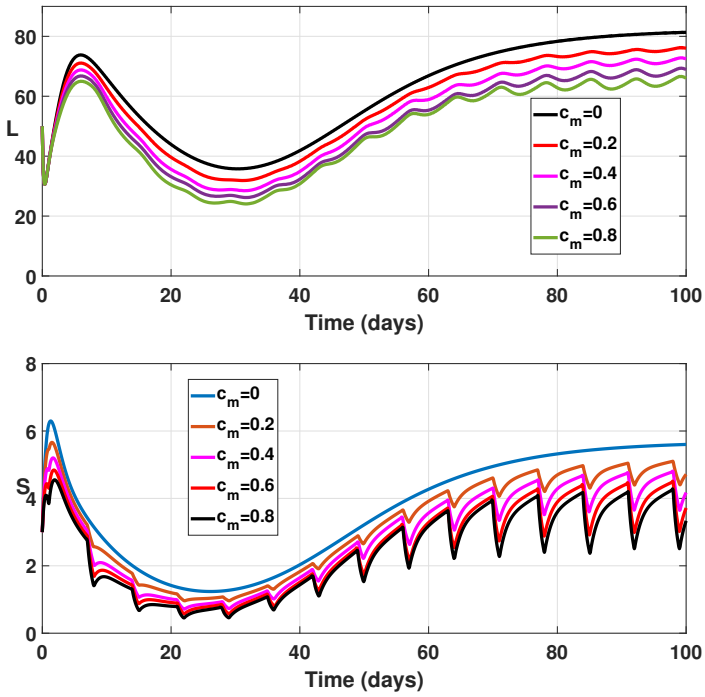

Figure 6: Effect of the use of insecticide on the population of larvae L and blood meal seeking mosquitoes S.

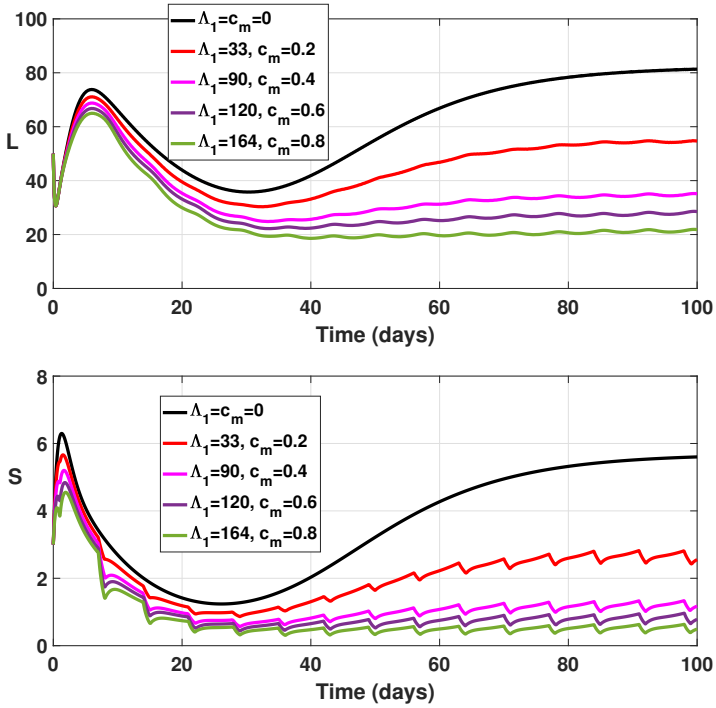

Figure 7: Effect of the use of insecticide combined with the release of sterile males mosquitoes on the population of larvae L and blood meal seeking mosquitoes S.

IV CONCLUSION

We analysed a complete life cycle model of mosquitoes, vectors of some diseases in human communities, which take into account the principal stage of the gonotrophic cycle of female mosquitoes. To control the population of eggs, larvae and female mosquitoes in blood meal seeking stage, we combined the sterile insect technique with the use of insecticides to kill female mosquitoes in blood meal seeking stage. To this end, we took into account a compartment of sterile male mosquitoes, and we modified the mortality rate of seeking mosquitoes. Unlike other published works in the literature which consider that sterile and wild mosquitoes have a same probability to mate with a wild female [9, 10], we introduce one parameter e to traduce the fact that female mosquitoes, before mating, can make a choice between wild male and sterile male mosquitoes. We prove the global stability of the trivial equilibrium and showed that when the mosquito reproductive number $\mathcal{N}^*$ is greater than a certain threshold $\mathcal{N}_c$ greater than one, the model admits two positive equilibria. This shows that, according to the quality and the quantity of the release of sterile males, the traditional condition $\mathcal{N}^* > 1$ is not sufficient to the proliferation of mosquito populations. So, the sterile insect technique has a great impact in the control of mosquito population.

To illustrate our analytical results, we used parameters of *Aedes species* and performed numerical simulations. Indeed, we have shown numerically existence of two positive equilibria whenever the mosquito reproduction number $\mathcal{N}^*$ is greater than one and $\mathcal{N}_c$ ($1 < \mathcal{N}_c \leq \mathcal{N}^*$). On the other hand, if this condition is violated, then the model with sterile mosquitoes does not have a non-trivial equilibrium. It was clear that if female mosquitoes have a preference to mate with sterile males, the population of larvae will decrease significantly. Also, we used the technique of "pulse control" which contrast with the non-realistic continuous controls and concluded that the combination of sterile insect techniques and insecticides have a great impact on the decrease of total number of larvae and female mosquitoes in blood meal seeking stage, and so, can permit to decrease the total number of females mosquitoes.

A USEFUL DEFINITIONS AND RESULTS.

First of all, we provide a few preliminaries for our proof.

Theorem 4: p. 112 of [15]
Assume that $f : \mathcal{U} \longrightarrow \mathbb{R}^n$ is cooperative, where $\mathcal{U}$ is open and convex, and that $y, z : [t_0, t_0 + a] \longrightarrow \mathcal{U}$ are differentiable. If $y(t_0) \leq z(t_0), \quad \dot{y}(t) \leq f(y(t)), \quad \dot{z}(t) = f(z(t)) \quad \forall t \in [t_0, t_0 + a]$, then $y \leq z$, on $[t_0, t_0 + a]$.

Theorem 5:
[14, p. 62] Assume that $\dot{y} = f(y)$ is cooperative in an open convex set $\mathcal{U} \subseteq \mathbb{R}^n$ and that y^0 and y^1, $y^0 < y^1$ are the only equilibria in $\mathcal{U}$. If the Jacobian matrix $Df(y^0)$ is irreducible and $s(Df(y^0)) := \max\{\mathcal{R}e\ \lambda : \lambda \text{ is an eigenvalue of } A\}$, then there exists a unique solution $y(t)$ (up to translation) satisfying $\dot{y}(t) > 0$, for all $t \in \mathbb{R}$, $y(t) \to y^0$, as $t \to -\infty$, and $y(t) \to y^1$, as $t \to +\infty$.

REFERENCES

[1] H. Abboubakar, J. C. Kamgang, L. N. Nkamba, D. Tieudjo, Bifurcation thresholds and optimal control in transmission dynamics of arboviral diseases, Journal of Mathematical Biology 76 (2018) 379–427.

[2] H. Abboubakar, J. C. Kamgang, D. Tieudjo, Backward bifurcation and control in transmission dynamics of arboviral diseases, Mathematical Biosciences 278 (2016) 100–129.

[3] R. Anguelov, Y. Dumont, J. Lubuma, Mathematical modeling of sterile insect technology for control of anopheles mosquito, Computers & Mathematics with Applications 64 (3) (2012) 374–389.

[4] S. Ai, J. Li, J. Lu, Mosquito-stage-structured malaria models and their global dynamics, SIAM Journal on Applied Mathematics 72 (4) (2012) 1213–1237.

[5] S. N. Arifin, Y. Zhou, G. J. Davis, J. E. Gentile, G. R. Madey, F. H. Collins, An agent-based model of the population dynamics of anopheles gambiae, Malaria journal 13 (1) (2014) 424.

[6] W. Beklemishev, Le cycle gonotrophique, principe de base de la biologie de an. gambiae, Vop Fiziol Ekol Malar Komara 1 (3).

[7] P. Carnevale, M.-F. Bosseno, M. Molinier, J. Lancien, F. Le Pont, A. Zoulani, Etude du cycle gonotrophique d'anopheles gambiae (diptera, culicidae)(giles, 1902) en zone de forêt dégradée d'afrique centrale, Cah ORSTOM sér Ent Med Parasitol 17 (1979) 55–75.

[8] K. L. Coon, K. J. Vogel, M. R. Brown, M. R. Strand, Mosquitoes rely on their gut microbiota for development, Molecular ecology 23 (11) (2014) 2727–2739.

[9] Y. Dumont, F. Chiroleu, Vector control for the chikungunya disease, Mathematical biosciences and engineering 7 (2) (2010) 313–345.

[10] Y. Dumont, J. Tchuenche, Mathematical studies on the sterile insect technique for the chikungunya disease and aedes albopictus, Journal of mathematical Biology 65 (5) (2012) 809–854.

[11] J. E. Gentile, S. S. Rund, G. R. Madey, Modelling sterile insect technique to control the population of anopheles gambiae, Malaria journal 14 (1) (2015) 92.

[12] J. P. LaSalle, The stability of dynamical systems, Vol. 25, Siam, 1976.

[13] D. Moulay, M. Aziz-Alaoui, M. Cadivel, The chikungunya disease: modeling, vector and transmission global dynamics, Mathematical biosciences 229 (1) (2011) 50–63.

[14] Hal L. Smith, Monotone dynamical systems: an introduction to the theory of competitive and cooperative systems, American Mathematical Soc (2008).

[15] W. Walter, Ordinary differential equations, Springer-Verlag.

[16] WHO, Global brief on vector-borne diseases, World Health Organization, Geneva.

[17] WHO, UNICEF, et al., Global vector control response 2017-2030, World Health Organization, Geneva.

[18] H. M. YANG, M. D. L. DA GRAÇA MACORIS, K. C. GALVANI, AND M. T. M. ANDRIGHETTI, *Follow up estimation of aedes aegypti entomological parameters and mathematical modellings*, Biosystems, 103 (2011), pp. 360–371.

[19] N. YUSOFF, H. BUDIN, AND S. ISMAIL, *Stage-structured population dynamics of aedes aegypti*, in International Journal of Modern Physics: Conference Series, vol. 9, World Scientific, 2012, pp. 364–372.

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